An improved voltage-collapse protection algorithm based on local phasors

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1. Abstract

An improved algorithm to protect against voltage collapse based on two consecutive voltage- and current-phasor measurements to evaluate the voltage stability at a bus is proposed. The change in the apparent-power bus injection during a time interval is exploited for computing the voltage-collapse criterion. The criterion is based on the fact that in the vicinity of the voltage collapse no extra apparent power can be delivered to the affected bus. The one-step algorithm needs no additional checking, irrespective of whether the lines are generating or consuming reactive power.

The new algorithm was rigorously tested on different test systems. The results were obtained on a static IEEE 30-bus test system and on the dynamic Belgian-French 32-bus test system that includes full dynamic models of all the power-system components crucial to a voltage-instability analysis. The results show the advantages of the proposed algorithm: it is simple, computationally very fast and easy to implement in a numerical relay.

Keywords: Local phasors; Apparent-power losses; Voltage collapse; Relay algorithm

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2. Introduction

The operation of power systems closer to their load limits is dictated by the needs of deregulated electricity markets. However, as a result, several blackouts have occurred due to voltage instability. This means that voltage stability has become a matter of serious concern for system operators and is the subject of considerable investigation because of its importance in terms of the security of the system and the quality of the power. Significant efforts are still being directed towards definitions, classifications, new concepts, practices and tools for solving the voltage-stability and security-analysis problems [1], [2].

The voltage-instability problem is characterized by uncontrollable voltages at certain locations in a power network after a disturbance. The problem occurs more frequently in stressed networks that have reduced stability margins and/or reduced reactive-power reserves. Voltage instability is basically a dynamic phenomenon with rather slow dynamics and a time domain ranging from a few seconds to some minutes, or more.

The main idea behind local methods is that the local phasors contain enough information to directly detect the voltage-stability margin. This concept is attractive since real-time measurements of the voltage and current phasors at the system buses are already available from the phasor measurement units (PMUs) installed in many power systems [3]. In addition to the benefits of a small computational effort and simplicity, local methods also give a very good insight into the nature of the voltage-collapse process and can easily be used for protection schemes.

The local method proposed by Strmčnik and Gubina [4] is based on power-system decomposition on the reactive-power radial transmission paths modelled as two-bus equivalents. In addition to the proximity to the voltage collapse, the method also considers the exhaustion of the generator's reactive-power reserves. However, a weakness of this method is the complex identification of the critical buses in the system and the fact that the electric distance of the reactive-power source from the affected load bus is not considered when calculating the generator's reactive-power reserve.

Vu. et al. [5] proposed a numerical relay based on the power-transfer impedance-matching principle. The measured data and a parameter-identification recursive algorithm are used to obtain the apparent impedance of the load and Thevenin's equivalent of the system as seen from the load bus. The voltage collapse occurs when these impedances are equal. The local relay can also be realized as a numerical relay based on tracking the voltage drop across the Thevenin's impedance.

Verbič and Gubina [6] proposed a numerical relay based on the fact that in the vicinity of the voltage collapse the entire increase in the apparent power loading at the sending end of the line is due to the supply of transmission losses. The main shortfall of this method is the additional checking to see if the line is loaded below its natural loading.

In this paper we propose an improved version of the algorithm proposed in [6]. The new algorithm deals with load currents instead of with the line currents at a bus, and therefore there is no additional checking to see whether the lines are generating or consuming reactive power. The practical applicability of the proposed algorithm and the real-time measurement issues are demonstrated on the dynamic Belgian-French 32-bus test system.

3. Theoretical background

Verbič and Gubina [6] proposed the S difference criterion (SDC), which is based on the fact that in the vicinity of the voltage collapse the stressed lines become consumers of large amounts of reactive power and they begin limiting the reactive-power supply to a load bus. In other words, all of the increase in the apparent power flow at the sending end of a line supplies the transmission losses. This means:

$$\Delta\underline{S}_{\mathrm{ki}} = \underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{ki}}^{*} + \underline{I}_{\mathrm{ki}}^{*}\Delta\underline{U}_{\mathrm{k}} + \Delta\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{ki}}^{*} = 0\,, \tag{1}$$

at the receiving end of a line i-j, which supplies bus k as in Fig. 1. $\underline{U}_{\mathrm{k}}$ is the voltage of bus k, which is supplied by the current $\underline{I}_{\mathrm{ki}}$. It has also been shown that this condition coincides with the singularity of the system's Jacobian matrix [7]. Neglecting the higher term in (1), $\Delta\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{ki}}^{*}=0$, the SDC index is defined as:

$$\mathrm{SDC}=\left|1+\underline{I}_{\mathrm{ki}}^{*}\Delta\underline{U}_{\mathrm{k}}/\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{ki}}^{*}\right|. \tag{2}$$

At the point of voltage collapse, when $\Delta\underline{S}_{\mathrm{ki}}=0$, the SDC equals 0 [6].

The condition $\Delta\underline{S}_{\mathrm{ki}}=0$ at the receiving end of a line results in a demand for a higher reactive-power inflow on the other lines connected to the affected bus k, and in the final step all of the connected lines fail to supply the load bus k [6]. Consequently, no additional apparent power can be delivered to the load k. This can be formally written as:

$$\Delta\underline{S}_{\mathrm{k}}=0. \tag{3}$$

In response to a disturbance the driving force for voltage instability is usually the restoration of the load power by under-load tap-changer (ULTC) transformers and the limiting of the generators field currents by the over-excitation limiters. This fact leads us to observe the load currents instead of the line currents.

Equation (3) can be further developed in a similar way to (1). Since $\Delta\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{k}}^{*}$ represents a very small value if a sufficiently small sample step is used, it can be neglected and (3) can be written as:

$$\Delta\underline{S}_{\mathrm{k}}=\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{k}}^{*}+\underline{I}_{\mathrm{k}}^{*}\Delta\underline{U}_{\mathrm{k}}=0, \tag{4}$$

or

$$\left|\underline{U}_{\mathrm{k}}\Delta\underline{I}_{\mathrm{k}}^{*}\right|=\left|-\Delta\underline{U}_{\mathrm{k}}\underline{I}_{\mathrm{k}}^{*}\right|, \tag{5}$$

and also

$$\left|\underline{U}_{\mathrm{k}}/\underline{I}_{\mathrm{k}}^{*}\right|=\left|\Delta\underline{U}_{\mathrm{k}}/\Delta\underline{I}_{\mathrm{k}}^{*}\right|, \tag{6}$$

where $\Delta\underline{U}_{\mathrm{k}}$ and $\Delta\underline{I}_{\mathrm{k}}^{*}$ represent the complex voltage and current increments for the bus k with respect to their base-case values $\underline{U}_{\mathrm{k}}$ and $\underline{I}_{\mathrm{k}}^{*}$. The increments are determined after the system is subjected to a set of disturbances in the system. With two consecutive online measurement samples of bus voltage and load current from a PMU at the bus k, $\Delta\underline{U}_{\mathrm{k}}$, and $\Delta\underline{I}_{\mathrm{k}}^{*}$, voltage and current differences can easily be obtained.

An improved SDC index based on the bus apparent-power difference criterion (BSDC) can be defined as:

$$\mathrm{BSDC} = \left(\left|\underline{U}_{\mathrm{k}} / \underline{I}_{\mathrm{k}}^{*}\right| - \left|\Delta\underline{U}_{\mathrm{k}} / \Delta\underline{I}_{\mathrm{k}}^{*}\right|\right) / \left|\underline{U}_{\mathrm{k}} / \underline{I}_{\mathrm{k}}^{*}\right| = 1 - \left(\left|\Delta\underline{U}_{\mathrm{k}} / \Delta\underline{I}_{\mathrm{k}}^{*}\right| / \left|\underline{U}_{\mathrm{k}} / \underline{I}_{\mathrm{k}}^{*}\right|\right). \qquad (7)$$

In the case of normal loading the simulation shows that the condition $\left|\underline{U}_{\mathrm{k}} / \underline{I}_{\mathrm{k}}^{*}\right| \gg \left|\Delta\underline{U}_{\mathrm{k}} / \Delta\underline{I}_{\mathrm{k}}^{*}\right|$ holds. At the voltage-collapse point, when (6) is fulfilled, the BSDC reduces to 0. This information can be easily used in a numerical protective relay. Note that (6) is also fulfilled when the system's operating point does not change, $\Delta\underline{U}_{\mathrm{k}} = 0$, and $\Delta\underline{I}_{\mathrm{k}}^{*} = 0$. This situation is not critical for the voltage stability and should be disregarded.

There is a small but important difference in the calculation when using the SDC and the BSDC: when using the BSDC no additional check as to whether a line is loaded under or over its natural loading has to be made. Moreover, the BSDC index directly detects the affected bus, while the SDC deals only with the affected lines. At the same time, the BSDC calculation reduces the number of measurements needed for a protection relay against a voltage collapse.

4. Relay algorithm

The flowchart of the proposed algorithm is shown in Fig. 2. A phasor measurement unit (PMU) provides synchronized measurements of the real-time phasors of the bus voltage and the load-bus currents. The algorithm uses cyclic measurement samples at the bus relay point. In the first step the voltage and load-current phasors are measured and their values are stored. Based on the last two consecutive measurements the difference between the currents and voltages at the last two time intervals are then calculated to detect the change in the operational state in the selected time interval.

Note again that $\Delta\underline{U}_{\mathrm{k}}$ and $\Delta\underline{I}_{\mathrm{k}}^{*}$ can only be identified if the system's operating point changes. If the last state was not critical with respect to a voltage collapse, the next state with no change is not critical for voltage stability either, and should therefore be disregarded. Dividing by 0 is also not allowed; therefore, the fraction $\left|\Delta\underline{U}_{\mathrm{k}} / \Delta\underline{I}_{\mathrm{k}}^{*}\right|$ is only calculated when the condition $\left|\Delta\underline{I}_{\mathrm{k}}\right| > \left|\Delta\underline{I}_{\mathrm{min}}\right|$ is fulfilled, otherwise it takes the preceding value, or the value 1.

Traditionally, the BSDC is calculated using (7) based on a series of consecutive phasor measurements by calculating the difference between the last two phasor values. Since in this paper, the phasors are obtained though simulations, the BSDC is calculated at every time increment, the length of which can be chosen according to the rate of change in voltage and current. The time increment can be reduced to very small values during the fast dynamics in the system, excited by discrete changes of the voltage set point, the tap position and the tripping of the line. During the BSDC oscillations its lowest value is considered and its newly calculated value is accepted if it is smaller than the previous one, which means that only the worst case is tracked.

When the calculated BSDC is smaller than the predefined threshold $\mathrm{BSDC}_{\mathrm{min}}$ the triggering signal is sent. It might trigger selected generators to increase their reactive power production, the nearest compensating devices that should be activated, selected transformers that should block their tap changers, or selected consumers at the affected bus in order that they shed their load [6].

5. Test results

The improved protection algorithm was tested on the static IEEE 30-bus test system and on the dynamic Belgian-French 32-bus meshed test system. The static test system was used to demonstrate the weakness of the SDC and how this is improved by using the BSDC. Furthermore, the dynamic test system with full dynamic models of the system components crucial to the long-term voltage stability phenomenon was used to show the practical applicability of the proposed algorithm.

5.1. Static tests with the IEEE 30-bus test system

Continuation power flow (CPF) [8] was used to determine the maximum deliverable power. The loads are modelled as constant-power loads, thus the maximum deliverable power coincides with the stability limit, as discussed in Section 2. Loading is simultaneously increased at all the load buses with a constant step of 1% and the limits of the reactive-power generation are taken into account. The load factor is assumed to be constant at all load levels.

The IEEE 30-bus test system has five P-V buses and twenty one P-Q buses. The total demand of the base case is 283.4 MW and 126.2 Mvar. Voltage collapse occurs at a load level of 433.5 MW.

The SDC for the observed line 17-10, which generates reactive power, is shown in Fig. 3. Fig. 3 reveals that the SDC is not a good indicator for determining the proximity of the voltage collapse for lines that are loaded with less than their natural loading. Although the system load is increased uniformly, the SDCs during normal loading (between 283.4 MW and 339 MW) have uncommon values: the SDC for the sending end of line 17-10 is extremely low and the SDC for the receiving end of line 17-10 takes values of more than 1. On the other hand, the BSDC at buses 10 and 17, in Fig. 4, does not have any problem and correctly determines the voltage-collapse trajectory.

The BSDCs in Fig. 4 have a somewhat steeper decline at the loading level between 283.4 MW and 339 MW. Load-flow results indicate that some P-V buses at these loading levels hit their reactive-power generation limits, and therefore their status changes to P-Q buses.

5.2. Dynamic tests by the Belgian-French 32-bus test system

The Belgian-French 32-bus test system displayed in Fig. 5 was first used in [9] and [10] to demonstrate the mechanisms leading to a voltage collapse. The same system was later used in [2] as a benchmark for a comparison of the voltage-stability indices proposed so far. A time-domain simulation was performed in the EUROSTAG. More details on the system modelling, the scenario, and the simulation results can be found in [10].

The system represents a simplified 400-kV and 150-kV Belgian-French system from the early 1980s. The external system is modelled by means of three infinite buses. Two important power stations, N1 and N10, produce most of the power. The global load of the system is mainly located at the sub-transmission level, downstream of the 150/70 kV ULTC transformers. The loads connected to buses N201-N207 consist of a mix of induction motors, constant impedance loads, and compensation capacitors. Buses N203 and N205 are located in the “southern” part of the system, and the others in the “northern” part.

This 32-bus meshed system undergoes a 2-hour load pick-up of 30%, followed by tripping of the north international tie line N16-N3 at 5000 s. The system transiently recovers from this “large” contingency. After 7400 s unit M2 trips for unknown reasons. Because of this trip and the action of the automatic tap changers, the rotor current limit of the M1 unit is reached. The limiter is badly tuned and drastically reduces the excitation, causing the loss of synchronism of the M1 unit about 2 minutes after M2 trips. After this second unit tripping, the voltage on the terminals of the other units is progressively reduced because of further tap changes and rotor-current limiter actions. The remaining generating units successively trip as a result of under the voltage protection or because of a loss of synchronism, leading to the final blackout of the system, as depicted in Fig. 6.

As we stated before, the BSDC can only be calculated if the system’s operating point changes, since a division by 0 is not allowed. Hence, the new BSDC is calculated if the load-current difference between two operating points, $\Delta I_{\min}$, is greater than the predefined threshold. The threshold magnitude is defined by the sensitivity of the BSDC to different changes in the system. The BSDC sensitivity increases with decrease of $\Delta I_{\min}$, and the algorithm gets more conservative.

A generalization of the $\Delta I_{\min}$ threshold should be adjusted to different test and real systems, which requires adequate testing.

The BSDC is calculated using (7) at every time increment, which in our case varies from 0.001 s to 35 s [10]. The threshold $\Delta I_{\min}$ used here was experimentally set to 0.015 p.u. The BSDC changes versus time for buses N204 and N205 are plotted in Fig. 7. The ULTCs, the voltage support device limits and the loss of the line N16-N3 produce sharp changes of the BSDC. In all cases, the index was normalized with respect to the initial values.

Observe in Fig. 7 that the bus N205 located in the "southern" part of the system is not severely affected by the tripping of the north international tie-line N16-N3 at 5000 s. The sharp changes at times 1500 s, 5214 s, 5277 s, 5499 s and 6100 s are due to the operation of the transformer ULTCs located in the "southern" part or the very nearby "northern" part of the system. At 7400 s, when the unit M2 trips, the load has already exceeded the maximum deliverable power of the system without units N16 and M2 [2]. The stability boundary is crossed, as indicated by BSDC = 0, and the system collapse progresses rapidly after this point. Just before the voltage collapse the BSDC is still relatively high due to reactive power support from the "southern" generators, and this bus is considered to be not critical. The shapes of the BSDCs for both the "southern" buses, N203 and N205, are very similar.

In contrast, the tripping of the line N16-N3 strongly affects the "northern" part of the system, as shown in Fig. 7, where the BSDC for the bus N204 indicates a reduction of the voltage-collapse margin. The BSDC is reduced to 0.2, which reveals that the bus N204 is a part of the critical "northern" area, which is also demonstrated in [2]. The smaller sharp changes are again due to the action of the transformer ULTCs.

The voltage-instability problem is characterized by the voltage uncontrollability at certain locations in the power system. Sharp reductions in the BSDC indicate a reduction of the collapse margin. Hence, the corrective actions, such as blocking of the tap changers, triggering of the voltage increase signals at the selected nearest generators or at the compensating devices, and load-shedding activation at the affected buses, can be directly associated with the sharp index changes. When corrective actions should be applied a BSDC threshold can be defined heuristically, based on the historical data, on the off-line stability, on contingency studies, and on an operator's experience. The results in Fig. 7 suggest that the BSDC threshold should be set to at least 0.65 or higher, depending on the desired tolerance level, in order to make the first additional corrective and/or coordinated actions in critical areas. If the BSDC reaches 0.2 or less, it is the last moment to shed the selected consumers at the affected bus in order to possibly avoid a system collapse.

6. Discussion

The condition (3) which defines the maximum deliverable power can be found in various forms in textbooks on basic circuit theory. Most indices based on that condition can only be implemented at the control centre with communication links to the substations. The advantage of the method presented in [5] and the proposed method is that they involve only local measurements; therefore they are tailored for a relay application. The new approach in this paper makes it further possible to simplify determination of the proximity to voltage collapse, which is calculated only from two consecutive phasor measurements and quite distinctly from the adaptive curve-fitting techniques used in [5].

The operating point where the load and network PV curves are tangent to each other coincides with the maximum deliverable power (3) only when the load is assumed to be of constant power. In voltage stability analysis, we focus on aggregated loads as seen by a bulk power delivery transformer, e.g. the load connected to bus k in Fig. 1. The long term constant power assumptions used in many fast voltage stability analyses are justified for many systems where ULTCs restore the aggregate load. For behaviour of the steady-state voltage-sensitive load, the loadability limit may not correspond to the nose point stability limit. For such cases, our proposed prediction is pessimistic.

Extensive field measurements reported in [12] have shown that aggregate loads become more sensitive to voltage variations in a transient period than in the steady-state. Therefore, in real power systems, the maximum deliverable power (3) becomes the loadability and stability limit. Past this limit there is a loss of equilibrium and a voltage collapse will occur progressively [2], [11], as shown in the dynamic test example in this paper.

Certain mechanisms of loss of equilibrium and stability in nonlinear dynamical systems are inherently connected to bifurcations. Local bifurcations, which have received most of the attention, are saddle-node, limit-induced and Hopf bifurcations, given their association with voltage and oscillatory stability problems [2], [11]. Hence, the prediction of these bifurcations, especially saddle-node through the use of a variety of indices has drawn significant attention [2], [13]-[14].

Indices based on first order information, such as the proposed BSDC, may be inadequate to predict possible instability problems in practical systems, which are strongly connected to the limit-induced and Hopf bifurcations. These bifurcations usually occur in the highly stressed systems due to large discontinuities in the presence of systems control limits (e.g. generator limits) or major line tripping. This problem can be reduced by considering a “second order” index, where the index is divided by its gradient with respect to the parameter under study. That exploits additional information embedded in these indices, as suggested in [2], [13]-[14].

7. Conclusion

An improved algorithm for protecting against voltage instability is presented in this paper. The algorithm only requires information about the voltage and current phasors in order to evaluate the system’s voltage stability at a bus in the form of a voltage-collapse prediction index. It represents a line of defence at the local level, as well as support of the system-wide protection against voltage collapse. The improved criterion has been tested on the static IEEE 30-bus test system and on the dynamic Belgian-French 32-bus test system, which undergoes an extensive system disturbance, to show the practical applicability of the proposed algorithm.

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Vitae

Ivan Šmon was born on March 1976 in Jesenice, Slovenia. He received his Diploma Engineer degree and the Ph. D. degree from the University of Ljubljana in 2001 and 2006, respectively. In 2001 he joined "Elektro Gorenjska" Distribution Company in Kranj for one year. Between 2003 and 2006 he was employed at the University of Ljubljana, Faculty of Electrical Engineering as a Research Assistant. His main research field includes power-system operation and control in new deregulated environment. Currently he is with the Ministry of the Economy, Directorate for Energy.

Miloš Pantoš was born in 1977 in Slovenia. He received the Diploma Engineer degree and the Ph.D. degree from the University of Ljubljana in 2001 and 2005, respectively. He is employed as an assistant professor at the University of Ljubljana, Faculty of Electrical Engineering. He is a head of Laboratory of Power Systems and High Voltage and Laboratory of Power Systems Protection and Remote Control. His main research field includes power-system operation, protection and control, power market and ancillary services.

Ferdinand Gubina was born in 1939 in Slovenia. He received Diploma Engineer, M.Sc., and Dr.Sc. degrees from the University of Ljubljana, Slovenia, in 1963, 1969, and 1972, respectively. From 1963 he was with “Milan Vidmar” Electroinstitute, Ljubljana, where he headed the Power System Operation and Control Department. In 1970 he joined the Ohio State University, Columbus, Ohio, for a year, as a teaching and research associate. Since 1988 he has been a full professor at the

University of Ljubljana. His main interests lie in the area of electric power system operation and control.

Figures

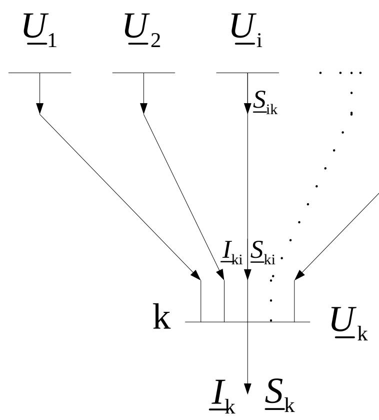

Fig. 1 Supplying of load bus k.

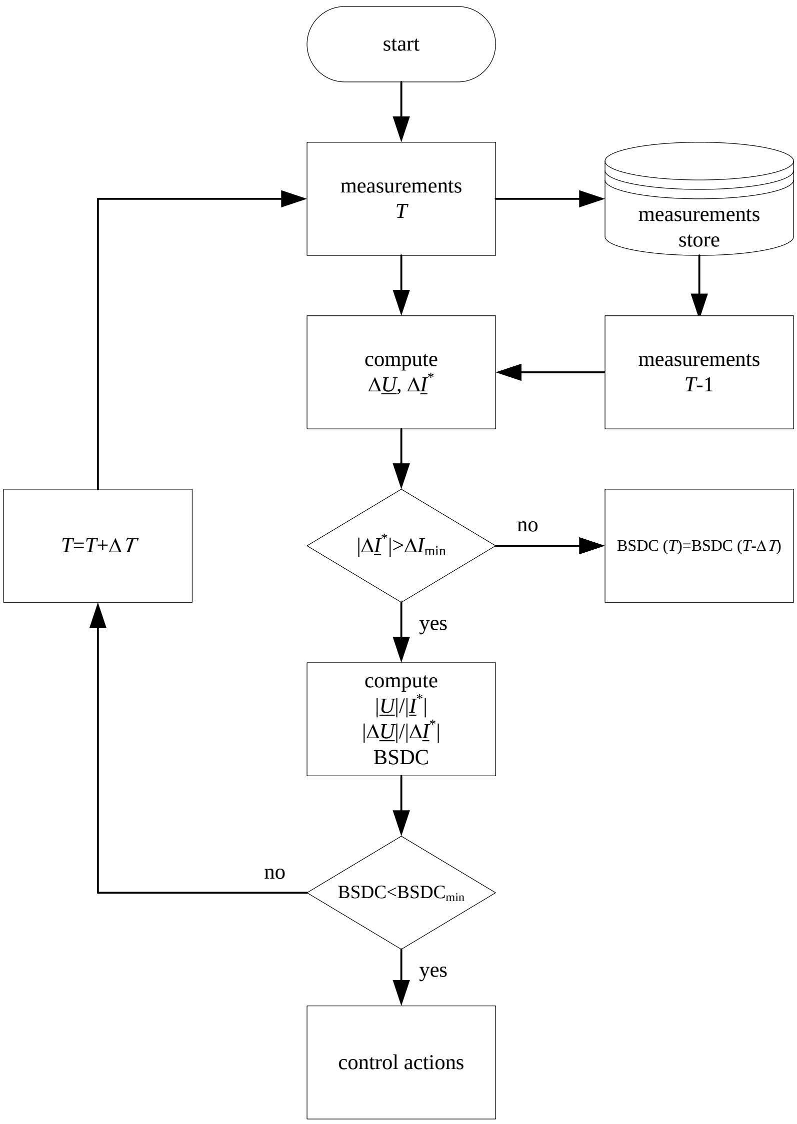

Fig. 2 The flowchart of the proposed algorithm.

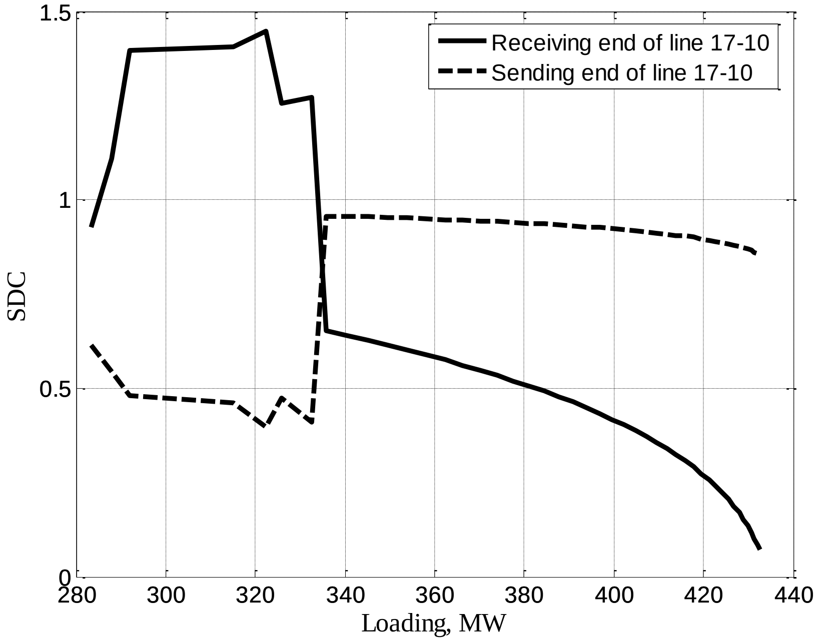

Fig. 3 IEEE 30-bus test system, SDC for line 17-10.

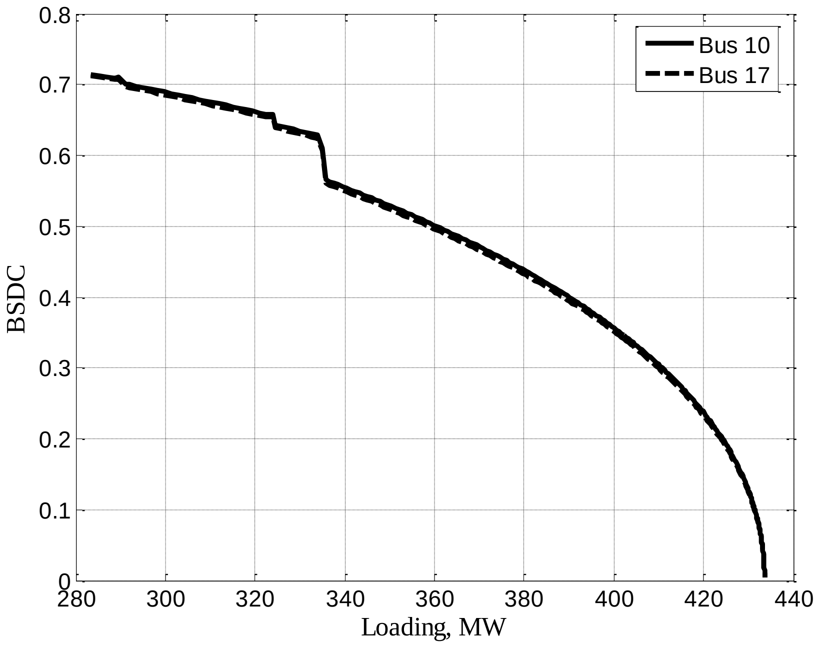

Fig. 4 IEEE 30-bus test system, BSDC for buses 10 and 17.

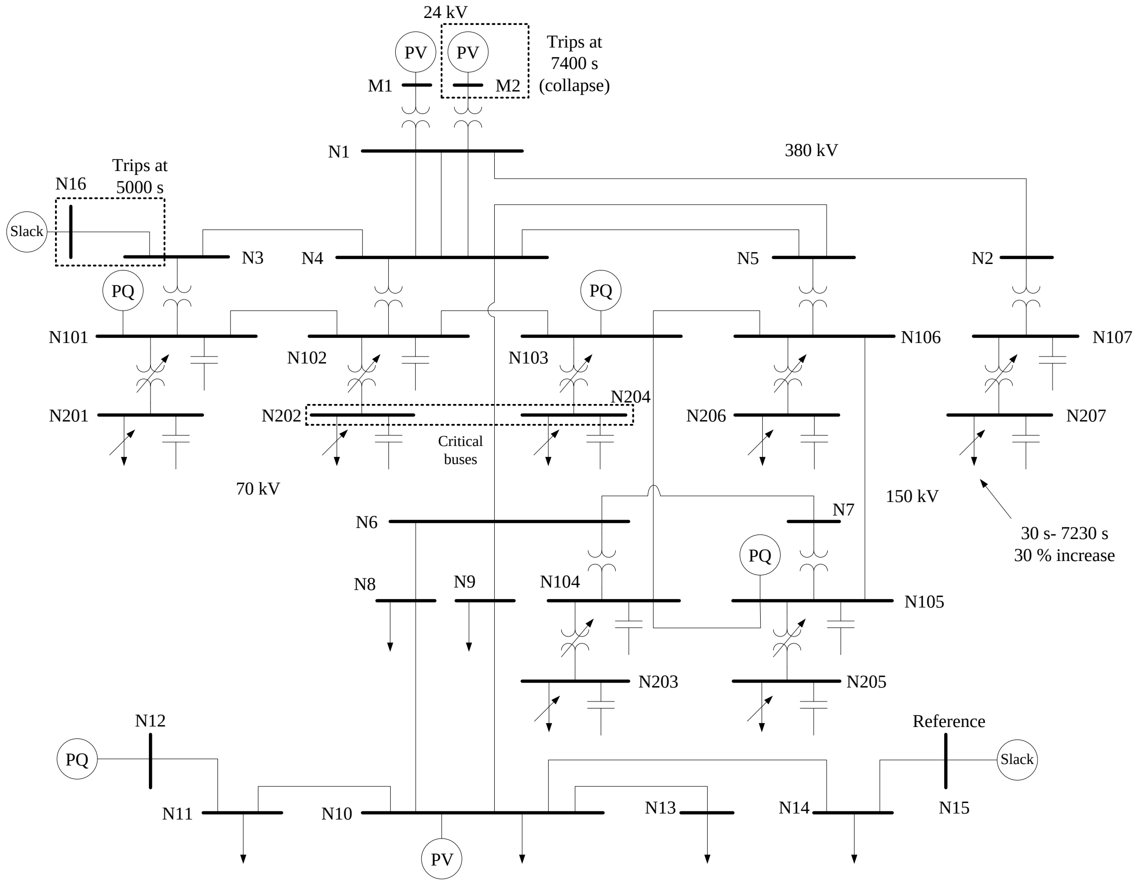

Fig. 5 Belgian-French 32-bus test system.

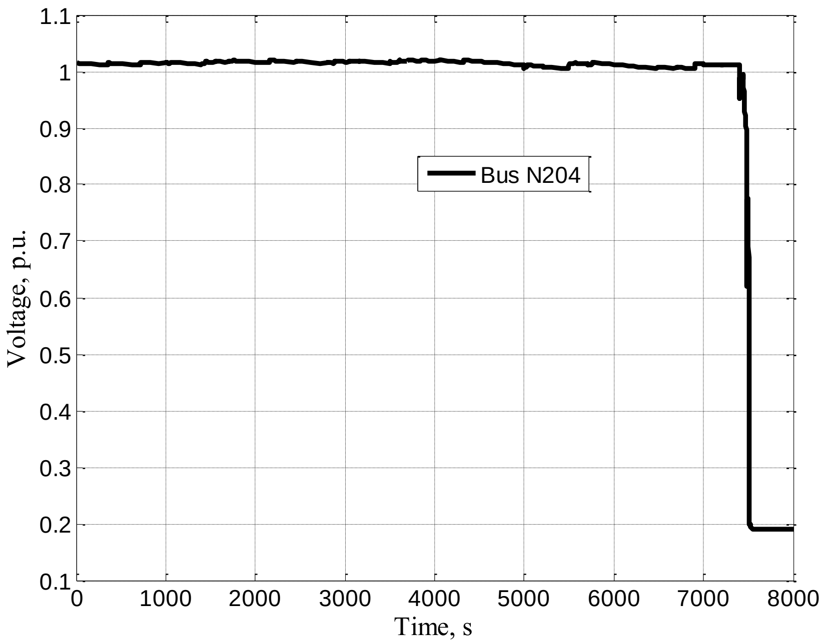

Fig. 6 Belgian-French 32-bus test system, voltage at bus N204.

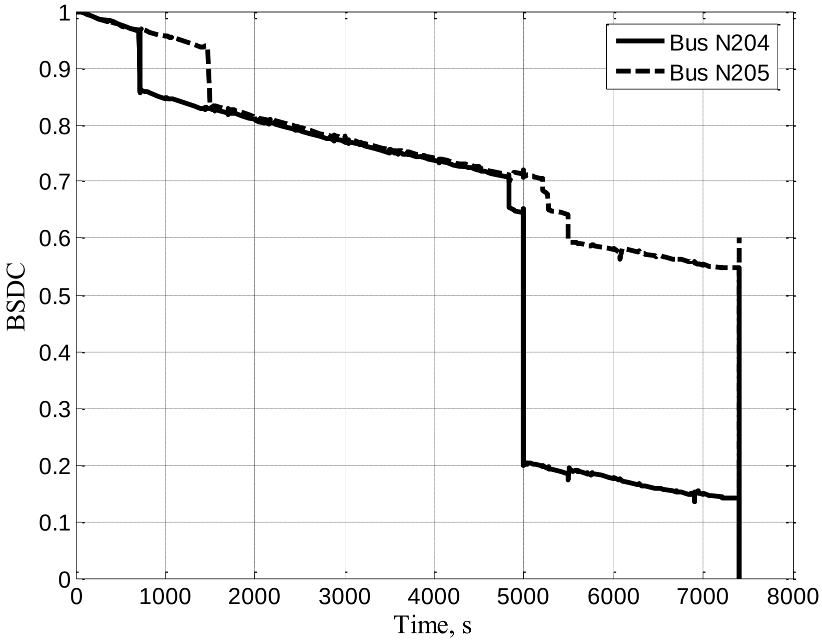

Fig. 7 Belgian-French 32-bus test system, buses N204 and N205, BSDC, $\Delta I_{\min} = 0.015\,\text{p.u.}$