Applying Transformer Limits in Power Flow Studies Using Unconstrained Optimizations

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***Abstract*—The letter proposes a new method which analyses the states of transformers in Power Flow studies, by directly considering the limits of both the transformer state (e.g minimum and maximum tap) and the control (e.g. minimum and maximum voltage). The method uses an empirically developed cost function, which rapidly increases, where the transformer limits are exceeded and decreases, where they are not. Unconstrained optimization techniques are used to minimize the cost function. Since the structural limits (e.g maximum or minimum tap) are stricter than the control limits (e.g. minimum and maximum voltage), the cost function is designed so that its increase rate for exceeding control limits is lower than if structural limits are exceeded. The method is developed for phase-shifting, tap-changing and quadrature-boosting transformers and is tested on the IEEE 300 bus system with six devices operating simultaneously.**

***Index Terms*— Power flow control limits, Power flow transformer modelling, Unconstrained optimizations.**

I. Introduction

THE presence of transformers in power flow (PF) studies creates new variables since their states are generally not known beforehand and are determined by their control criteria. The operational manner of transformers is that they alter their state (e.g. their tap) to keep the controlled variable (e.g. voltage magnitude) between a minimum and a maximum value. Since transformer states are physically restricted, the variable reflecting their state is also restricted to a limited interval.

The general mechanism for incorporating transformers in PF studies is to set the control criterion to a specific value, and then find the corresponding transformer states. The way in which the new transformer states are calculated is by solving an equation system, where the transformer states are the unknown variables, and the transformer control criteria provide the equations. As the underlying PF algorithm dictates the numerical approach of the calculation, different techniques must be applied for different PF algorithms. For the Newton-Raphson type PF, for example, the method in [1] is applicable, while for the Holomorphic Embedding type PF, the method in [2] must be used instead.

The technique of modelling the transformers by producing their own equations, has the necessity, that the control criteria of the transformers are strictly defined as a single dimensional value. This may cause issues when applied to a real system. The reason for this is that the control criteria can only be strictly specified to a single value, when the user, knows that a controlled variable has exceeded its limits, which requires running a PF. Likewise, the control criteria can only be foregone (and the device replaced with a fixed transformer) if calculations show that the device structural limits would otherwise be exceeded for a given specified control. In essence, the problem with the established transformer analysis techniques is that whenever a limit, be it a structural or a control limit, is reached, the underlying system of equations is changed and must be solved again.

In this paper, a method which uses unconstrained optimization techniques to model transformers by accounting for the limits directly, is presented. The method construct a cost function (CF), which has the property that it is always positive, and rapidly increases in case a transformer limit is exceeded. Since control and structural limits are not equally important (the control limits may be respected only for as long as the structural limits are respected) the CF has a different structure for the two types of limits. The considered transformer types are the On-Load Tap Changer (OLTC), the Phase Shifting Transformer (PST) and the Quadrature Boosting Transformer (QBT).

The CF is illustrated in Fig. 1 for the case of a single PST operating in the IEEE 300 bus system. The rectangle indicates the transformer control and phase limits, and the circles represent the possible solutions for an illustrative case.

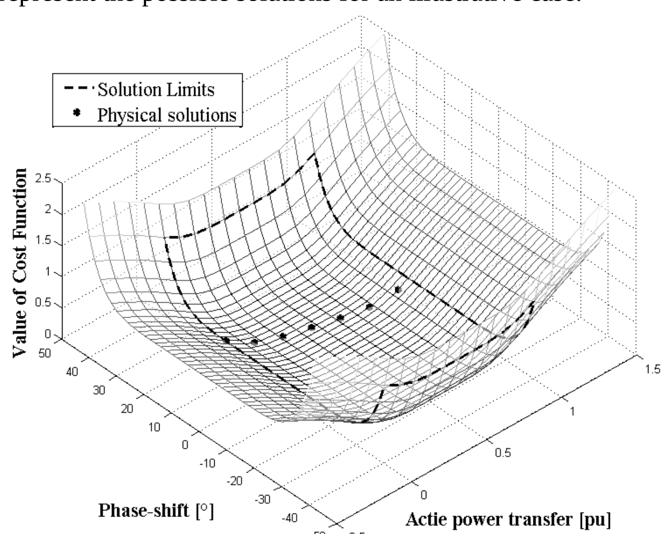

Fig. 1. Solution illustration.

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In the rest of the letter, the CF construction is detailed in Section II. and numerical test results are in Section III. The conclusions drawn from this letter are in Section IV.

II. Method Development

The CF is a dimensionless function obtained by summing partial CF elements of each transformer. Each transformer has two CF elements, one corresponding to the structural and the other to the control criterion of the device. For sake of generality, let us denote the transformer state with γ, which is the tap of an OLTC (t_{OLTC}), the phase-shift of a PST (φ_{PST}), or the relative value of quadrature injected voltage of a QBT (c_{QBT}); and the control variable with ψ, which is the voltage magnitude for the OLTC (V_{OLTC}), or the active power flow for the PST (P_{PST}) and QBT (P_{QBT})). The CF can be expressed as:

$$CF = \sum_{\mathrm{i}=1}^{N} \xi_{\gamma-\mathrm{i}} + \sum_{\mathrm{i}=1}^{N} \xi_{\psi-\mathrm{i}} \,, \tag{1}$$

where N is the number of all transformers and $\xi_{\gamma\text{-i}}$ and $\xi_{\psi\text{-i}}$ are the CF elements that correspond to the structural and control limits of the i-th transformer respectively.

The first part of the *CF* uses the even-order power function (EOPF). The higher the power of the EOPF, the quicker it will rise if the absolute value of the argument is above 1, and the quicker it will fall towards zero if the absolute value is below 1. The CF element $\xi_\gamma(\gamma)$, which operates on γ, is given in (2), where n is an arbitrary integer used to control the sharpness, and $\gamma_{\min}$ and $\gamma_{\max}$ are the minimum and maximum bounds respectively.

$$\xi_\gamma(\gamma) = \left(\frac{\gamma - (\gamma_{\max} + \gamma_{\min})/2}{(\gamma_{\max} - \gamma_{\min})/2} \right)^{2n} \tag{2}$$

When building the part of the CF that corresponds to the control criterion, $\xi_\psi(\psi)$, we must acknowledge that the device will maintain the controlled variable within limits only if permitted by the structural limits. If the device cannot keep the controlled variable within limits, it will forego the control and stay at the minimum or maximum structural limit.

To embody the above described in the CF we lean on the bell-shaped membership-function (BSMF) [3]. An inverted BSMF (obtained by subtracting the BSMF from 1) is multiplied by an EOPF structured as in (2). The order of this EOPF is lower than that used in (2). We establish this by using another arbitrary integer m, which must be lower than n. The lower order of the EOPF used for $\xi_\psi(\psi)$ makes it have less of an impact outside of min-max limits of ψ in the total CF as compared to the $\xi_\gamma(\gamma)$, while multiplication with the inverted BSMF makes $\xi_\psi(\psi)$ fall quickly to zero if ψ is within limits.

$$\xi_\psi(\psi) = K_{\psi 1}(\psi) \cdot K_{\psi 2}(\psi) \,, \tag{3}$$

where:

$$K_{\psi 1}(\psi) = \left(\frac{\psi - (\psi_{\max} + \psi_{\min})/2}{(\psi_{\max} - \psi_{\min})/2} \right)^{2m} \tag{4}$$

$$K_{\psi 2}(\psi) = 1 - 1 \Bigg/ \left(1 + \left(\frac{2\psi - \psi_{\max} - \psi_{\min}}{\psi_{\max} - \psi_{\min}} \right)^{2\cdot(n-m)} \right) \tag{5}$$

$$m < n \qquad m, n \in \mathbb{Z} \qquad m, n \geq 0 \tag{6}$$

Fig. 2 illustrates how the developed cost function elements, $\xi_\gamma(\gamma)$ and $\xi_\psi(\psi)$, change with the parameters n and m.

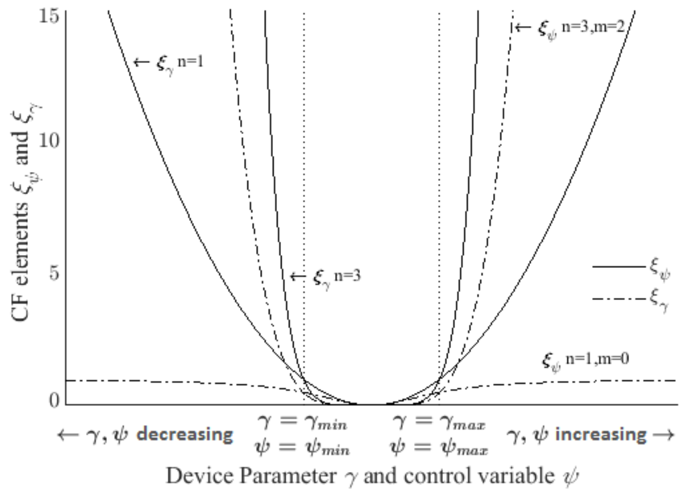

Fig. 2. Cost function elements $\xi_\gamma(\gamma)$ and $\xi_\psi(\psi)$.

A. Applying the QBT to PF data

The PF programs take the transformer taps and phases directly as their input data, which makes applying the state of OLTC and PST devices straightforward. The quadrature injected voltage of QBT devices, however, does not have a direct representation. The states of QBT devices (c_{QBT}) must be expressed in terms of transformer tap and phase as shown in (7) and illustrated in Fig. 3.

$$t_{\mathrm{QBT}} = \sqrt{c_{\mathrm{QBT}}^2 + 1} \quad , \quad \varphi_{\mathrm{QBT}} = \arctan\left(c_{\mathrm{QBT}}\right) \tag{7}$$

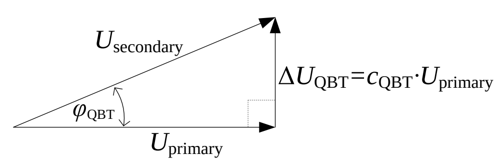

Fig. 3. Phasor diagram of QBT.

B. On deciding the parameters n and m

The performance of the method is dependent upon arbitrary parameters n and m. Deciding the value for these parameters is discussed next. It should be noted that parameter n influences the impact of structural constraints of the transformers on the CF, while m influences the impact of control criteria. Therefore, if for a specific calculation we find that the structural limits are exceeded, n is to be increased.

Increasing only parameter n, however, may make the optimization algorithm overlook the effect of the control criteria. In such a case, m must also be increased so that the effect of the function $\xi_\psi(\psi)$ in the CF is emphasized.

Setting m and n is case specific. Increasing one with respect to the other will emphasize one CF element with respect to the other. In our tests, the typical values used for *m* and *n* were within the region of a few tens, well below one hundred.

C. Physically attainable state of transformers

There are two key aspects to mention regarding the solution with the proposed method. The first aspect is related to the fact that the proposed method operates by assuming continuous transformer states. This is not related to the real-world case, where the transformers have discrete tap settings attainable. After the solution is obtained, it is therefore recommended that an additional PF is ran where the transformer states are set to the closes physically attainable ones.

Aside from the above relation to the discrete world, there is also an additional consequence of using unconstrained optimization techniques for enforcing constraints. In order for some element of the CF to start rapidly increasing, the respective variable (either the state of transformer or the controlled variable) must be exceeding its limits. Therefore, in order for the structural variables γ to have a relatively large impact on the CF, they have to exceed their bounds, albeit slightly. The consequence of this fact is that, when transformers are unable to perform their designated control, and therefore reach their structural limits, their structural variables γ are found to be slightly outside of the min-max bounds. A case where this happens is illustrated in Section III.B. A final PF must therefore be ran where the transformers which exceed their bounds have their states set to the closest physically attainable state—at the respective limit.

III. Numerical Examples

The method was tested on the IEEE 300 bus system by inserting two transformers of each type (2 OLTCs, 2 PSTs, 2QBTs). Table 1 shows the transformer locations as well as the base-case values, present in the original IEEE 300 bus system. In the optimization approach, the PF software that was ran as a subroutine was MatPower [4].

Two cases were performed. In the first case, the structural limits were designed so that they did not interfere with the control, and all six transformers were enabled to fulfill their control criteria. In the second case, one transformer of each type had its structural limits more stringently set, so that the corresponding control could not be fulfilled and the method had to bring the respective device to its structural limit. The initial conditions of both cases were set to a flat start, i.e. the taps are set to 1 pu and the phase-shifts to 0°. In both cases, the following values for parameters *n* and *m* were used:

$$n = 12 \quad , \quad m = 3 \tag{8}$$

Table 1. Location of transformers and base-case values, IEEE 300 bus system.

Device	OLTC 1	OLTC 2	PST 1	PST 2	QBT 1	QBT 2
Branch	100	310	150	350	200	250
P_{base} [pu]	/	/	1.405	-1.907	0.995	-0.274
V_{base} [pu]	1.0216	1.0180	/	/	/	/

A. Case 1 – Soft structural constraints

In this case, the device structural limitations were set beyond what we may encounter in practice, so that only the control criteria are effective in the CF. The device parameters and the solution data for this case are shown in Table 2.

Table 2. Transformer control and state limits and obtained solution for case 1.

	γ_{min}	γ_{max}	P_{min} or V_{min} [pu]	P_{max} or V_{max} [pu]	γ_{final}	P_{final} or V_{final} [pu]
OLTC 1	0.95	1.25	1.05	1.10	1.234	1.0572
OLTC 2	0.95	1.10	1.05	1.10	1.076	1.0548
PST 1	-20.0	20.0	0.00	1.00	12.95	0.3917
PST 2	-70.0	20.0	-1.00	0.00	-62.13	-0.0018
QBT 1	-0.60	0.60	-2.00	-1.00	0.579	-1.0039
QBT 2	-0.30	0.30	4.00	5.00	-0.274	4.0391

B. Case 2 – Stringent structural constraints

In this case, the transformer structural limits were set to more realistic values. The structural limits in this case affected the control capabilities, and some transformers were unable to carry out their desired control. The bounds of transformer states and control, as well as the solutions, are shown in Table 3.

Table 3: Transformer control and state limits and obtained solution for case 1.

	γ_{min}	γ_{max}	P_{min} or V_{min} [pu]	P_{max} or V_{max} [pu]	γ_{final}	P_{final} or V_{final} [pu]
OLTC 1	0.95	1.05	1.05	1.10	1.051	1.0298
OLTC 2	0.95	1.10	1.05	1.10	1.071	1.0527
PST 1	-20.0	20.0	0.00	1.00	9.93	0.6294
PST 2	-20.0	20.0	-1.00	0.00	-21.61	-1.5332
QBT 1	-0.30	0.30	-2.00	-1.00	0.308	-0.7212
QBT 2	-0.30	0.30	4.00	5.00	-0.275	4.0547

IV. Conclusions

This letter proposes a method, which directly implements limits of transformer-type devices (OLTC, PST and QBT), by using an unconstrained optimization approach, to minimize an empirically developed cost function. Since an unconstrained optimization approach is used, the method provides information about transformer states using a single run of an iterative procedure. The method was verified on the IEEE 300 bus system with 6 transformers (2 OLTCs, 2 PSTs and 2 QBTs) operating at once.

V. References

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