Modified Topological Generation and Load Distribution Factors

M. Pantoš, G. Verbič, F. Gubina

***Abstract*—In the paper, a modification of the Topological Generation Load Distribution Factor method of power flow tracing is reported. The modification is needed since the existing method introduces additional, fictitious nodes on system lines in order to take into account the transmission losses. Due to the expansion of a system, the existing algorithm based on the augmented matrix equation requires more mathematical effort and memory and longer calculation time to obtain a solution. New approach successfully avoids the matrix expansion by introducing matrix decoupling, which is its main improvement. The second novelty is an introduction of equivalent model of a line that unites the nodal reactive power with the power produced by shunt admittances. Thus, the modified method can be also applied to the reactive power flows and transmission-loss allocation.**

***Index Terms*—Modified Topological Generation Distribution Factor, Modified Topological Load Distribution Factor, power-flow tracing, power system**

I. Introduction

UNBUNDLING of the transmission service brings new requirements to the power system operation and power system control. In the energy market, the financial flow follows agreements among consumers and producers, while the power physically flows to the consumer from the nearest producer over the shortest electrical distance in compliance with the Kirchhoff's Laws. At first sight, the question "which producer supplies a particular consumer?" in a transmission network seems to be a very complicated one; however, with the requirement for open access to networks and the need to follow the network flows and losses in order to remunerate the network owners in a transparent and fair way, this question needs to be answered.

A few methods of power flow tracing were already proposed. The Domain Generation Distribution Factor (DGDF) method [1]-[3] is based on a generator's domain, i.e. a set of buses supplied by the same set of generators. The obtained clusters are viewed as new nodes, connected together with tie branches. Such a simplification leads to a significant change in the generator's contribution to some line flows due to a slight change in the system's topology. The Nodal Generation Distribution Factor (NGDF) method [4], [5] is based on a search algorithm that needs additional algorithm in case of circular flows, thus it is a time consuming method.

The Topological Generation and Load Distribution Factor (TGDF and TLDF) method [6]-[8] is based on the proportional-sharing principle that is neither provable nor disprovable. However, the method seems to be the most appropriate approach to the power-flow tracing since it is based on matrix calculation of producers' and consumers' shares in a line flow. It obtains a solution in an analytical manner without any simplifications and additional algorithms for loop power flows. The procedure introduces additional, fictitious nodes on system lines in order to take into account the transmission losses. Thus, augmented matrices lead to more complex calculation that requires longer computational time and more memory.

To overcome these problems, the paper presents several improvements of the TGDF (TLDF) method that enable tracing of active and reactive power. The proportional-sharing principle does not have to be stressed since the procedure is based on completely reasonable mathematical derivations.

The main improvement of the proposed solution is a matrix decoupling that enables consideration of transmission losses without any matrix extension, thus the computational speed and required memory are preserved. Other novelties of the proposed modification are equivalent model of a line, which is important for reactive power allocation, and calculation of nodal distribution factors of generators and loads that can be applied to the reactive-power management and market [9].

The improved TGDF (TLDF) method was tested on a simple 5-bus test system [10] to show its advantages. In general, the power-flow tracing methods can be efficiently used for transmission congestion management, reactive power management and transmission service pricing.

II. The Proposed Decoupled Tracing Method

To present the new improved approach to power-flow tracing, a short introduction of the TGDF (TLDF) method is required.

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A. The TGDF (TLDF) Method in Brief

Fig. 1 demonstrates the conditions at the node i, where G_i is the production, D_i is the consumption, Ψ_i is the set of nodes that directly supply the node i, and Ξ_i is the set of nodes that are directly supplied by the node i. The symbols S_{ia} and S_{ai} are the active or reactive-power flows on the line i-a, directed from the node i to the node a. Since the particular line flow changes because of a transaction loss $L_{ia} = S_{ia} - S_{ai}$, the values S_{ia} and S_{ai} are different. Hence, S_{ia} is the value at the node i, and S_{ai} is the value at the node a.

For each node, the total nodal flow S_i can be defined as the sum of all the inflows (1) or outflows (2) from the node i:

$$S_i = \sum_{b \in \Psi_i} S_{ib} + G_i \quad i = 1, 2, \ldots, n , \tag{1}$$

$$S_i = \sum_{a \in \Xi_i} S_{ia} + D_i \quad i = 1, 2, \ldots, n , \tag{2}$$

where n is the total number of nodes in the system.

In the subsequent text, the TGDF approach identifies the flow paths from the producers to the consumers by using (1). It calculates generators' shares on the lines. Similarly the TLDF approach derives the loads' shares on the lines from (2).

Before a derivation of TGDFs and TLDFs, the transmission losses have to be taken into the account. First possibility proposed by Bialek [6] is to use an average line flow on a certain line by adding half of the line loss to the power injection at the terminal node of that line. Similar solutions presented in [6] are to use a gross flow that is obtained by adding the line loss to the nodal demand at the terminal node and to use a net flow [6] by modifying the nodal generation while leaving the nodal demand unchanged. Since the reactive-power loss of a line may be quite considerable when compared with the flow itself, the proposed solutions do not obtain satisfactory results. Thus, additional, virtual nodes [8] can be added in the middle of each line. Each node acts as a reactive-power source or sink depending on the operating condition of the line. Fig. 2 presents this idea for the line i-a, where the power consumption or production in case of reactive power is equal to the transmission loss L_{ia}. It is also possible to use this approach for the active-power research to obtain more accurate results. Since the active power losses are always positive, all additional nodes in the system act as the sinks.

After the transmission losses are taken into the account, (1) can be reformulated to obtain a matrix notation:

$$\mathbf{A} \cdot \mathbf{S} = \mathbf{G} , \tag{3}$$

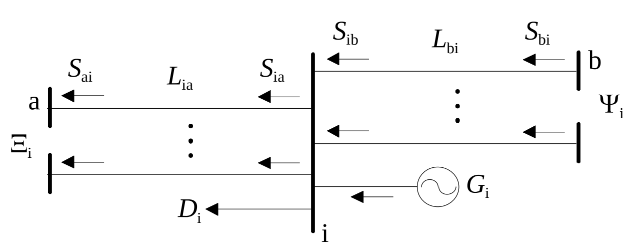

Fig. 1. Conditions at the node i.

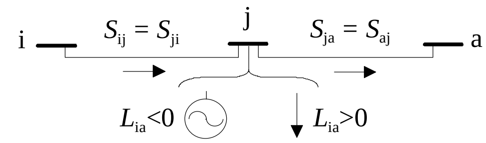

Fig. 2. Additional node on the line i-a.

where **A** is the distribution matrix, **S** is the unknown vector of nodal flows and **G** is the vector of nodal productions. Note that **A** is the sparse and non-symmetric matrix with its (i,b)-th element a_{ib}, which is equal to:

$$a_{ib} = \begin{cases} 1 & i = b \\ -S_{ib}/S_b & b \in \Psi_i ; i \neq b \\ 0 & b \notin \Psi_i ; i \neq b \end{cases} . \tag{4}$$

Due to introduction of additional nodes in order to allow for transmission losses, the dimension of the distribution matrix **A** is enlarged to (n+p)×(n+p) where n is the number of system nodes and p is the number of additional nodes, i.e. the number of lines in a system. Also, the vectors **S** and **G** are enlarged to the dimension (n+p).

Assuming that the inverse distribution matrix $\mathbf{T} = \mathbf{A}^{-1}$ exists, then the i-th element of vector **S** is defined as:

$$S_i = \sum_{k=1}^{n+p} t_{ik} G_k \quad i = 1,2,\ldots,(n+p) , \tag{5}$$

where t_{ik} is the (i,k)-th element of the matrix **T**. By using the proportional-sharing principle [6]-[8], the line power flow S_{ij} can be presented as:

$$S_{ij} = \frac{S_{ij}}{S_j} S_j = \frac{S_{ij}}{S_j} \sum_{k=1}^{n+p} t_{ik} G_k = \sum_{k=1}^{n+p} d_{ij,k} G_k , \tag{6}$$

for $j \in \Xi_i$. Furthermore, the share of generator k on the line i-j can be derived as:

$$d_{ij,k} = S_{ij} \frac{t_{ik}}{S_i} . \tag{7}$$

Note that the proportional-sharing principle has not been used at the derivation of (6). However, it is completely reasonable from the mathematical point of view since it yields in essence the same results as proportional-sharing procedures [5]-[8].

TGDF (TLDF) method for the power-flow tracing introduces additional nodes on the lines in order to take into account the transmission losses as well. The approach can be improved to avoid the matrix expansion, which requires longer computational time and more memory. The modifications are presented in the subsequent text.

B. Equivalent Model of a Line

For the reactive-power-research purposes, the improved TGDF (TLDF) method introduces the equivalent Π model of a line, Fig. 3. Although the transmission losses of reactive power depend on the line charging, it is possible to displace the reactive powers G_{Qi} and G_{Qa} produced by shunt admittances $B_{sh/2,ia}$:

$$G_{\mathrm{Qi}} = U_{\mathrm{i}}^2 B_{\mathrm{sh/2,ia}}\,, \tag{8}$$

$$G_{\mathrm{Qa}} = U_{\mathrm{a}}^2 B_{\mathrm{sh/2,ia}}\,, \tag{9}$$

into the nearby nodes with an assumption that the voltages of the shunt admittances are equal to the nearby nodal voltages U_{i} and U_{a}, respectively. The nodal voltages can be obtained by the power flow calculation, by measurements or could be approximated as 1 p.u. (per unit).

Besides the production of reactive power, the line i-a has the reactive power transmission loss L_{ia} at the reactance X_{ia}:

$$L_{\mathrm{ia}} = I_{\mathrm{ia}}^2 X_{\mathrm{ia}}\,, \tag{10}$$

where I_{ia} is the current on the line i-a. The equivalent model of a line unites the nodal reactive-power flows and reactive-power flows produced by shunt admittances, which results in the “isolation” of transmission losses on the reactance X_{ia}. This notion enables decoupling of the power flows as the improved approach suggests in the subsequent text.

C. Decoupled Line Power Flow

Identification of the generators’ and loads’ shares starts with line-power-flow decoupling presented in Fig. 4 for the line i-a. This line is decoupled into two lines transmitting lossless power flow S_{ai} and L_{ia}, separately. The flow balances at the nodes i and a are preserved, thus the proposed approach is reasonable from the mathematical point of view.

It is important to note that this solution is appropriate for the active-power flows since the transmission losses are always positive, thus the additional nodes act as the sinks. The reactive-power research requires previous introduction of the equivalent model of a line that insulates the line losses by the proposed displacement of reactive power sources into the terminal nodes, Fig. 3. The losses on the line reactances are always positive and proposed line power flow decoupling is therefore achievable.

In contrast to the Bialek’s solution with additional nodes in the middle of the lines, the described novelty also introduces fictitious nodes, but it enables matrix decoupling that yields the main improvement of the proposed modification. It is presented in the subsequent text.

D. Matrix Decoupling

On the basis of decoupled power flows, Fig. 4, (3) can be rewritten to show the new dimensions of the matrices:

$$\mathbf{A}_{(\mathrm{n+p})\times(\mathrm{n+p})} \cdot \mathbf{S}_{(\mathrm{n+p})\times 1} = \mathbf{G}_{(\mathrm{n+p})\times 1}\,, \tag{11}$$

where n is the number of original nodes and p is the number of additional, fictitious nodes in the system. Matrices **A**, **S** and **G** can be further decoupled as:

$$\begin{bmatrix} \mathbf{A}'_{\mathrm{n}\times\mathrm{n}} & \mathbf{0}_{\mathrm{n}\times\mathrm{p}} \\ \mathbf{A}''_{\mathrm{p}\times\mathrm{n}} & \mathbf{I}_{\mathrm{p}\times\mathrm{p}} \end{bmatrix}_{(\mathrm{n+p})\times(\mathrm{n+p})} \cdot \begin{bmatrix} \mathbf{S}'_{\mathrm{n}\times 1} \\ \mathbf{S}''_{\mathrm{p}\times 1} \end{bmatrix}_{(\mathrm{n+p})\times 1} = \begin{bmatrix} \mathbf{G}'_{\mathrm{n}\times 1} \\ \mathbf{0}_{\mathrm{p}\times 1} \end{bmatrix}_{(\mathrm{n+p})\times 1}, \tag{12}$$

which could be rewritten as:

$$\mathbf{A}' \cdot \mathbf{S}' + \mathbf{0} \cdot \mathbf{S}'' = \mathbf{G}'\,, \tag{13}$$

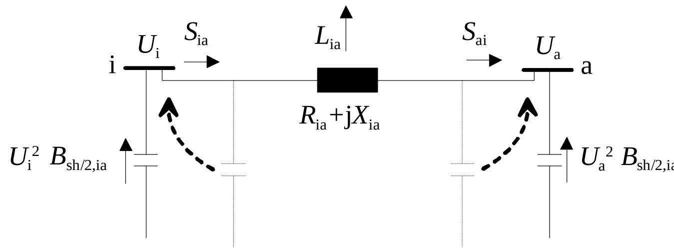

Fig. 3. Equivalent model of the line i-a.

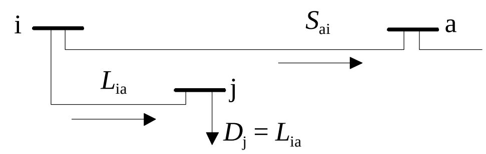

Fig. 4. Decoupled power flow of the line i-a.

$$\mathbf{A}'' \cdot \mathbf{S}' + \mathbf{I} \cdot \mathbf{S}'' = \mathbf{0}\,. \tag{14}$$

The distribution submatrix $\mathbf{A}'$ describes relations among original nodes and the submatrix $\mathbf{S}'$ is comprised of their nodal flows. The submatrix $\mathbf{A}''$ includes the elements related to the power flows on additional lines and nodal flows through additional nodes that are equal to the transmission losses and are also captured in the submatrix $\mathbf{S}''$. Since the additional lines transmit only losses from the original nodes to the additional nodes, the distribution submatrix $\mathbf{A}''$ carries information about the system losses. The submatrix $\mathbf{0}$ encompasses zero elements since all power flows on additional lines are directed from the original nodes to the additional nodes. Since the additional nodes are not mutual connected by the transmission lines, the submatrix $\mathbf{I}$ is the uniform matrix. The submatrix $\mathbf{G}'$ presents the productions at the original nodes.

To show the structure of the described submatrices, a simple 4-bus test system [6]-[8] with power flows in Fig. 5 is introduced. Power-flow decoupling can modify it as proposed in Fig. 4, which leads to the new flows presented in Fig. 6.

The expanded matrix equation (11) can be decoupled as follows:

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{S_{\mathrm{ba}}}{S_{\mathrm{a}}} & 1 & 0 & 0 \\ -\frac{S_{\mathrm{ca}}}{S_{\mathrm{a}}} & 0 & 1 & -\frac{S_{\mathrm{cd}}}{S_{\mathrm{d}}} \\ -\frac{S_{\mathrm{da}}}{S_{\mathrm{a}}} & -\frac{S_{\mathrm{db}}}{S_{\mathrm{b}}} & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} -\frac{L_{\mathrm{ab}}}{S_{\mathrm{a}}} & 0 & 0 & 0 \\ -\frac{L_{\mathrm{ac}}}{S_{\mathrm{a}}} & 0 & 0 & 0 \\ -\frac{L_{\mathrm{ad}}}{S_{\mathrm{a}}} & 0 & 0 & 0 \\ 0 & -\frac{L_{\mathrm{bd}}}{S_{\mathrm{b}}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{L_{\mathrm{dc}}}{S_{\mathrm{d}}} \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} S_{\mathrm{a}} \\ S_{\mathrm{b}} \\ S_{\mathrm{c}} \\ S_{\mathrm{d}} \end{bmatrix} \\ \begin{bmatrix} L_{\mathrm{ab}} \\ L_{\mathrm{ac}} \\ L_{\mathrm{ad}} \\ L_{\mathrm{bd}} \\ L_{\mathrm{dc}} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} G_{\mathrm{a}} \\ G_{\mathrm{b}} \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}, \tag{15}$$

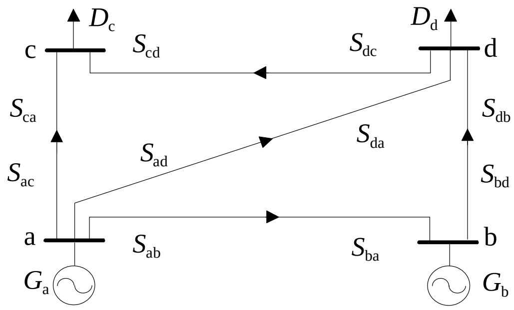

Fig. 5. 4-bus test system.

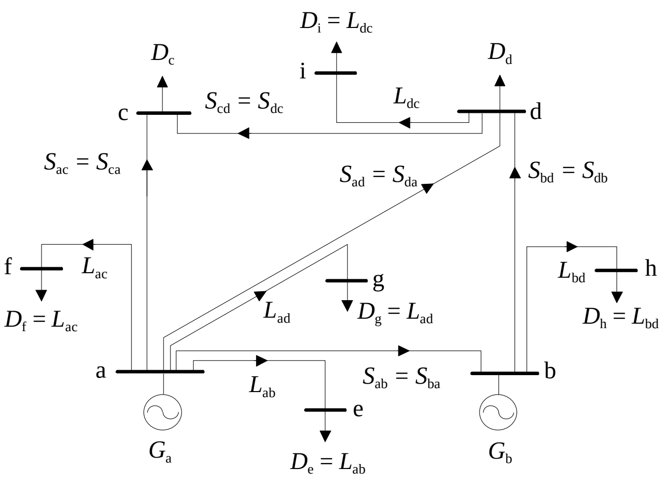

Fig. 6. Decoupled 4-bus test system.

which leads to (13) and (14). Since (14) gives trivial solutions, (13) is used to calculate generators' shares. It takes the form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{S_{ba}}{S_a} & 1 & 0 & 0 \\ -\frac{S_{ca}}{S_a} & 0 & 1 & -\frac{S_{cd}}{S_d} \\ -\frac{S_{da}}{S_a} & -\frac{S_{db}}{S_b} & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_a \\ S_b \\ S_c \\ S_d \end{bmatrix} = \begin{bmatrix} G_a \\ G_b \\ 0 \\ 0 \end{bmatrix}, \tag{16}$$

and it can be rewritten as (3). The generators' shares on the lines can be calculated using (7). With proposed matrix decoupling, the improved modification of the TGDF method requires less mathematical effort since the matrices' dimensions remain unchanged.

E. Generation Distribution Share Factor

To express more exactly, $d_{ij,k}$ is the share of the k-th generator's production in the power flow of the line i-j. It is also possible to reformulate the solution as:

$$LGDF_{ij,k} = d_{ij,k} \frac{G_k}{S_{ij}} = t_{ik} \frac{G_k}{S_i}, \tag{17}$$

where t_{ik} is the element of the inverse distribution matrix $\mathbf{T} = \mathbf{A}'^{-1}$ and $LGDF_{ij,k}$ is the Line Generation Distribution Factor, i.e. the share of the power flow on the line i-j that is produced by the generator at the node k. The main difference between these two notations can be seen from the following expressions:

$$\sum_{k=1}^{r} d_{ij,k} G_k = S_{ij}, \tag{18}$$

$$\sum_{k=1}^{r} LGDF_{ij,k} = 1, \tag{19}$$

where r is the total number of generators sharing the line load. In both cases, the sum of all generators' shares equals the total line power flow.

However, LGDFs are constant along lines in contrast to the shares in (7). This advantage of LGDFs is presented by a simple 2-bus test system in Fig. 7 with the generator at the node i producing 100 MW and the load at the node j consuming 98 MW. The line loss is equal to 2 MW. Table I shows the generator's shares on both ends of the line i-j. Since LGDFs are constant along the lines, they enable fair allocation of transmission losses, which is presented in appendix.

The improved method also obtains the generators' participation in the consumers' supply. The Generation Share Factor $GSF_{j,k}$ that represents the share of the k-th generator in the j-th consumption can be calculated as:

$$GSF_{j,k} = \frac{\sum_{i \in \Psi_j} LGDF_{ij,k} \cdot S_{ji}}{D_j}. \tag{20}$$

Like the GDFs, the GSFs are always positive, they take values between 0 and 1, and the sum of all the generators' shares in a specific consumption node is equal to 1.

$$0 \le GSF_{j,k} \le 1, \tag{21}$$

$$\sum_{k=1}^{r} GSF_{j,k} = 1, \tag{22}$$

where r is the total number of generators supplying the load at the node j. Nodal shares can be applied to the reactive-power management as proposed in [9].

F. Line Load Distribution Factor and Load Share Factor

The Line Load Distribution Factors LLDFs and Load Share Factors LSFs can be obtained by developing (2). Introduction of equivalent model of a line, Fig. 3, and line decoupling, Fig. 4, lead to the extended matrix notation:

$$\mathbf{B}_{(n+p)\times(n+p)} \cdot \mathbf{S}_{(n+p)\times 1} = \mathbf{D}_{(n+p)\times 1}, \tag{23}$$

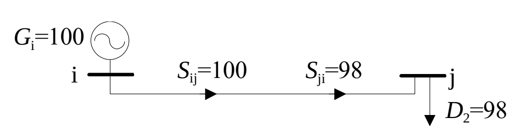

Fig. 7. Simple 2-bus test system with the active power in MW.

TABLE I
GENERATOR'S SHARES ON THE LINE I-J OF THE SIMPLE 2-BUS TEST SYSTEM

Line	Shares of generator i			
	$d_{ij,i}$	$d_{ji,i}$	$LGDF_{ij,i}$	$LGDF_{ji,i}$
i-j	1.00	0.98	1.00	1.00

where **D** is the vector of nodal consumptions and **B** is the distribution matrix with its (i,a)-th element b_{ia}, which is equal to:

$$b_{ia} = \begin{cases} 1 & i = a \\ -S_{ai}/S_a & a \in \Xi_i; i \neq a \\ 0 & a \notin \Xi_i; i \neq a \end{cases}, \quad (24)$$

where Ξ_i is the set of nodes that are directly supplied by the node i.

Equation (24) can be further decoupled as:

$$\begin{bmatrix} \mathbf{B}'_{n\times n} & \mathbf{B}''_{n\times p} \\ \mathbf{0}_{p\times n} & \mathbf{I}_{p\times p} \end{bmatrix}_{(n+p)\times(n+p)} \cdot \begin{bmatrix} \mathbf{S}'_{n\times 1} \\ \mathbf{S}''_{p\times 1} \end{bmatrix}_{(n+p)\times 1} = \begin{bmatrix} \mathbf{D}'_{n\times 1} \\ \mathbf{D}''_{p\times 1} \end{bmatrix}_{(n+p)\times 1}, \quad (25)$$

which could be rewritten as:

$$\mathbf{B}' \cdot \mathbf{S}' + \mathbf{B}'' \cdot \mathbf{S}'' = \mathbf{D}', \quad (26)$$

$$\mathbf{0} \cdot \mathbf{S}' + \mathbf{I} \cdot \mathbf{S}'' = \mathbf{D}''. \quad (27)$$

The distribution submatrices $\mathbf{B}'$ and $\mathbf{B}''$ are constructed similar as $\mathbf{A}'$ and $\mathbf{A}''$ according to (24). The submatrix $\mathbf{D}'$ presents the loads at the original nodes and $\mathbf{D}''$ includes the loadings at the additional nodes that are equal to the transmission losses, Fig. 4. Reformulation of (26) gives:

$$\mathbf{B}' \cdot \mathbf{S}' = \mathbf{D}' - \mathbf{B}'' \cdot \mathbf{S}'', \quad (28)$$

which could be rewritten as (23). According to the proportional-sharing principle [5]-[8], which can be omitted as it is concluded from the derivation in (6), and assuming that the inverse distribution-matrix $\mathbf{H} = \mathbf{B}^{-1}$ exists, the LLDFs and LSFs can be obtained as follows:

$$LLDF_{ij,k} = h_{ik} \frac{D_k}{S_i}, \quad (29)$$

$$LSF_{i,k} = \frac{\sum_{j\in\Xi_i} LLDF_{ij,k} \cdot S_{ij}}{G_i}, \quad (30)$$

where h_{ik} is the element of the inverse distribution matrix **H** and $LLDF_{ij,k}$ is the Load Distribution Factor, i.e. the share of the load k in the power flow of the line i-j. $LSF_{i,k}$ is the Load Share Factor that represents the share of the k-th load in the i-th production unit.

Since the modified TLDF approach treats the transmission losses as the loads, they also have their LLDFs on the lines. Instead of exercising (25) the loss share on the lines could be calculated as it is presented for loss share $LLDF_{ij,loss}$ on the line i-j:

$$LLDF_{ij,loss} = 1 - \sum_{k=1}^{w} LLDF_{ij,k}, \quad (31)$$

where w is the total number of loads. In this way, the actual and exact use of the network by a consumer is obtained.

G. Improved Algorithm for Power-Flow Tracing

To present the improved TGDF (TLDF) method more clearly, the algorithm that can be applied to the generators and loads is described in Fig. 8. After calculation of the power flows, active- or reactive-power-flow tracing can be performed. For the reactive-power research, the equivalent Π model of a line in Fig. 3 is introduced. Otherwise, the algorithm directly applies line-power-flow decoupling presented in Fig. 4 that is necessary for matrix decoupling in the next step before the final generators' or loads' shares are calculated.

III. Results

The advantage of the proposed modification of the TGDF (TLDF) method reflects in a faster calculation with less computer memory required, thus the comparison with the original TGDF method was obtained by the calculation of several inverse distribution matrices of different sizes.

Since only a relatively small set of different-sized models of power systems is available, the exact comparison can not be performed. Consequently, the construction of the distribution matrices was based on the assumption that an average power system has approximately 1.7-times more lines than nodes. This factor was heuristically determined. In this way, the distribution matrices were auto created using random values between 0 and 1 according to (4) that were randomly allocated within the matrices. Since the inversion of the distribution matrix **A** presents the most pretentious mathematical operation performed, the comparison was focused on time and memory needed just for calculation of the inverse distribution matrices and not the final results, i.e. the generators' and loads' shares. Figs. 9 and 10 present the results of both comparisons, respectively. Since the matrices are sparse, the technique of inverting sparse matrices was used.

For the chosen factor 1.7, the improved approach is approximately 9.2-times faster than the original method and needs 7.3-times less computational memory. From the results in Fig. 11, it can be seen that obtained ratios remain relatively steady despite of increasing size of power systems. However, the factor influences on the calculation time and memory required to obtain the solution by the original TGDF (TLDF) method since it is based on the extended matrices. Higher the factor is more computational time and memory is required to invert the distribution matrix and obtain the final results.

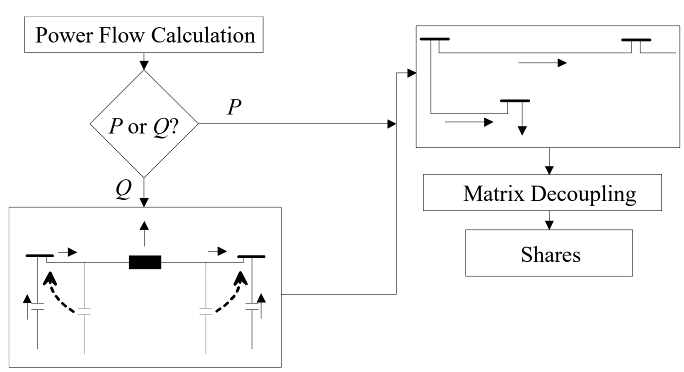

Fig. 8. Algorithm of the proposed method of power-flow tracing.

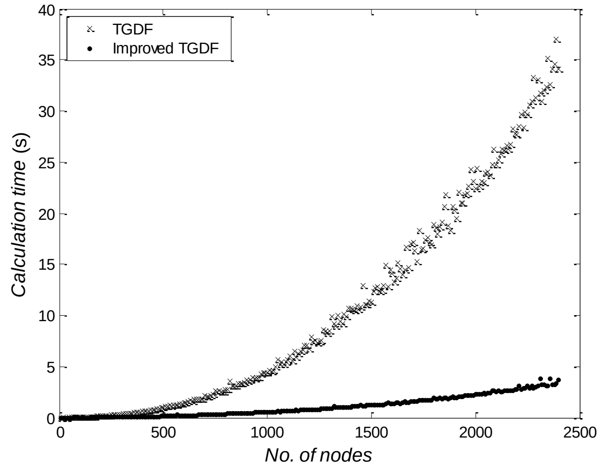

Fig. 9. Required calculation time.

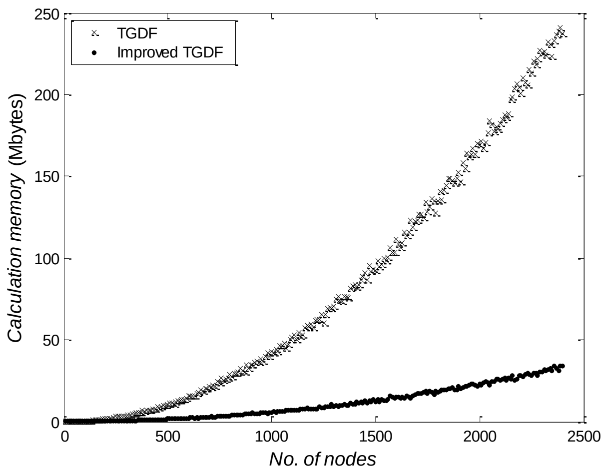

Fig. 10. Required calculation memory.

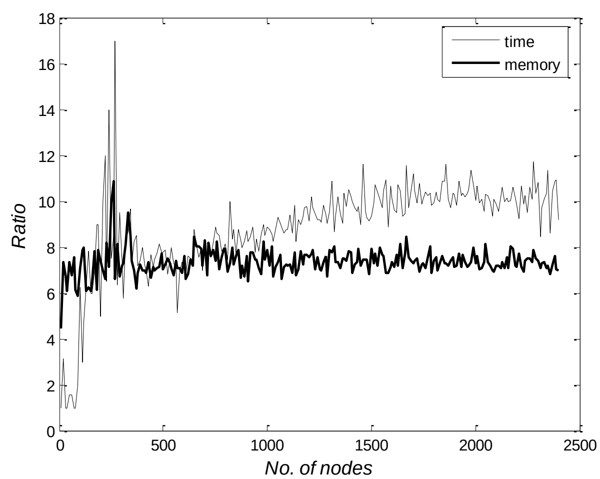

Fig. 11. Calculation time and memory ratio.

Although the structures of obtained matrices can be very different form structures of typical power systems, the procedure enables an illustrative comparison of the methods. For more precise results, real models of power systems should be considered.

To show that the new approach gives the comparable results to the TGDF method, the 5-bus test system [10] was considered in the subsequent text. Figs. 12 and 13 show the active- and reactive-power flows with the nodal voltages, respectively.

For the active power, the TGDF (TLDF) method and the modified approach allow for the losses in different ways presented in Figs. 14 and 15, respectively.

For the reactive power, the TGDF (TLDF) method applies the same methodology as for the active power, which is not explicitly shown in the paper. However, the improved TGDF method introduces the equivalent model of a line in Fig. 3 and the reactive-power flows are modified as it is presented in Fig. 16. The line reactive-power flows are further decoupled by the same approach as for the active power in Fig. 15, which is presented in Fig. 17.

Tables II and III present the generators' shares of the TGDF method and its improvement, respectively. Since two power systems with different configuration are compared, Table IV shows on which lines the TGDFs and LGDFs should be assessed by pairs.

The results in Tables II and III show, that both methods give the same results for the active power and different but comparable values of the reactive-power shares. The difference is caused by the introduction of the equivalent model of a line in Fig. 3 that modifies the reactive-power flows in the modified TGDF approach.

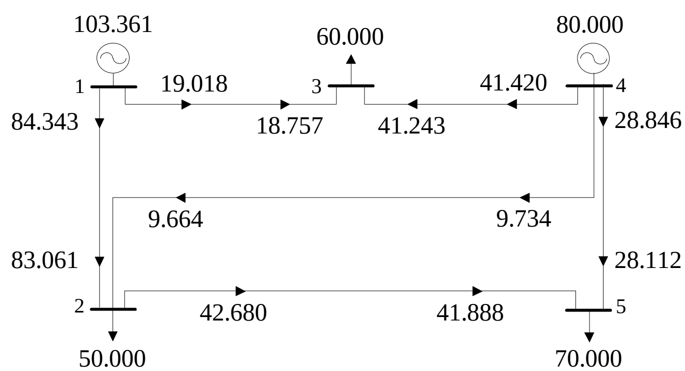

Fig. 12. 5-bus test system with the active-power flows in MW.

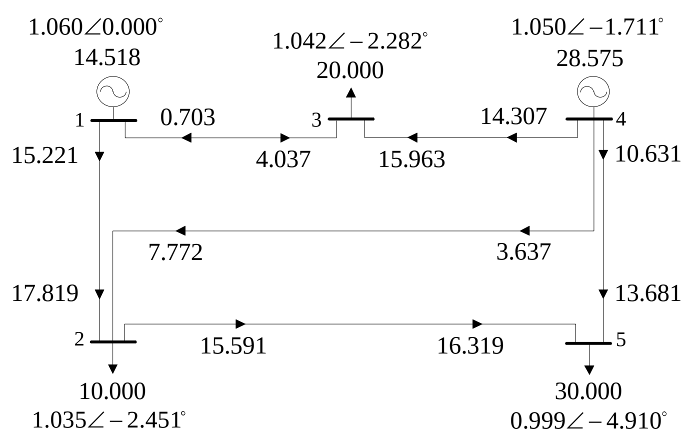

Fig. 13. 5-bus test system with the nodal voltages and the reactive-power flows in MVAr.

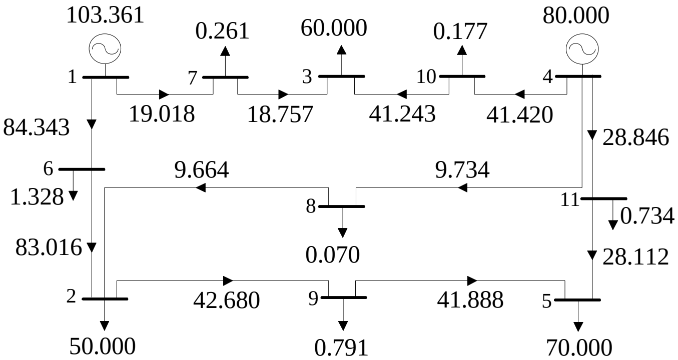

Fig. 14. 5-bus test system with the active-power flows in MW modified by the TGDF method.

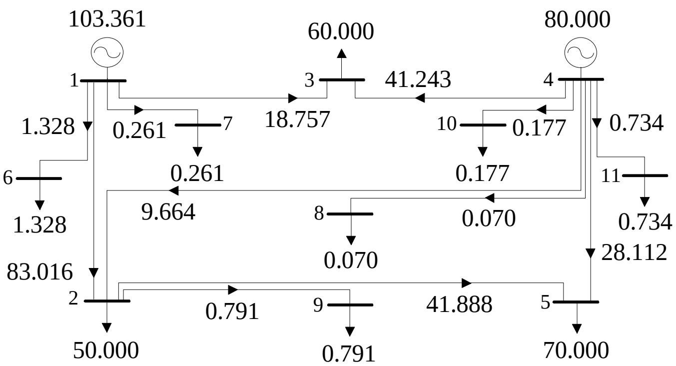

Fig. 15. 5-bus test system with the active-power flows in MW modified by the improved TGDF method.

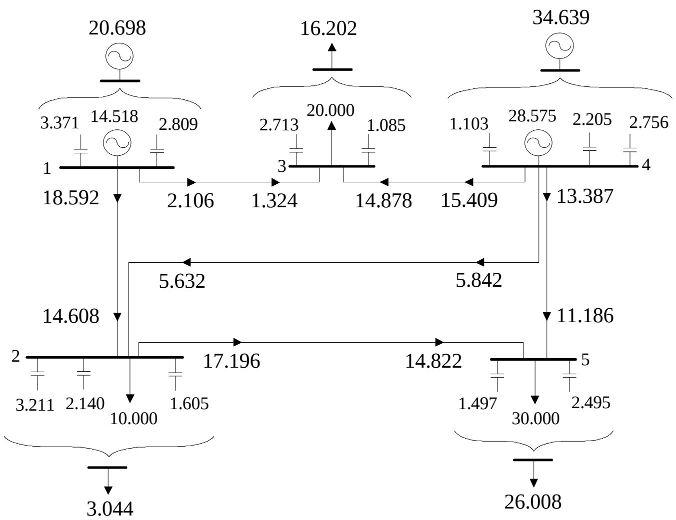

Fig. 16. Reactive power flows in MVAr after introducing the equivalent model of a line.

For a better comparison of the methods, the reactive-power shares of generators 1 and 4 on the lines are calculated for different system loadings. *K* is the ratio between the actual system loading and the base-case loading in Figs. 12 and 13. From the results in Tables V and VI it can be concluded that the methods give more comparable results when the system loading is higher. This conclusion is obvious since the system production decrease with the loading and it is neglectable comparing to the actual generator productions with higher shares on the lines.

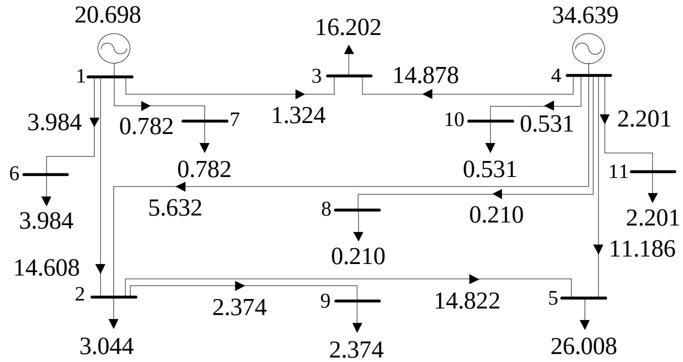

Fig. 17. 5-bus test system with the reactive-power flows in MVAr modified by the improved TGDF method.

TABLE II
TGDFs on the Lines Calculated by the TGDF Method

Line i-j	Active power		Reactive power	
	$TGDF_{ij,1}$	$TGDF_{ij,4}$	$TGDF_{ij,1}$	$TGDF_{ij,4}$
1-6	1.0000	0.0000	0.9538	0.0000
6-2	1.0000	0.0000	0.8148	0.0000
1-7	1.0000	0.0000	0.0000	0.0000
7-3	1.0000	0.0000	0.0000	0.0000
2-8	0.0000	1.0000	0.0000	0.4680
8-4	0.0000	1.0000	0.0000	1.0000
2-9	0.8957	0.1043	0.5673	0.1421
9-5	0.8957	0.1043	0.5420	0.1358
3-10	0.0000	1.0000	0.0000	0.8962
10-4	0.0000	1.0000	0.0000	1.0000
4-11	0.0000	1.0000	0.0000	1.0000
11-5	0.0000	1.0000	0.0000	0.7771

TABLE III
LGDFs on the Lines Calculated by the Improved TGDF Method

Line i-j	Active power		Reactive power	
	$LGDF_{ij,1}$	$LGDF_{ij,4}$	$LGDF_{ij,1}$	$LGDF_{ij,4}$
1-2	1.0000	0.0000	0.7014	0.0000
1-6	1.0000	0.0000	0.7014	0.0000
1-3	1.0000	0.0000	0.7014	0.0000
1-7	1.0000	0.0000	0.7014	0.0000
2-4	0.0000	1.0000	0.0000	0.8249
8-4	0.0000	1.0000	0.0000	0.8249
2-5	0.8957	0.1043	0.5063	0.2295
2-9	0.8957	0.1043	0.5063	0.2295
3-4	0.0000	1.0000	0.0000	0.8249
10-4	0.0000	1.0000	0.0000	0.8249
4-5	0.0000	1.0000	0.0000	0.8249
4-11	0.0000	1.0000	0.0000	0.8249

TABLE IV
Comparison of the TGDFs and LGDFs

	Lines					
TGDFs	6-2	7-3	2-8	9-5	3-10	11-5
LGDFs	1-2	1-3	2-4	2-5	3-4	4-5

IV. Conclusion

In the paper, the modified TGDF method for the power-flow tracing is reported. The proposed approach introduces several novelties of which a different consideration of the transmission losses is the most important since it enables decoupling of the extended matrices.

TABLE V
TGDFs for the Reactive Power for Different System Loading K

Line i-j	$TGDF_{ij,1}$ K = 1.2	$TGDF_{ij,1}$ K = 1.4	$TGDF_{ij,1}$ K = 1.6	$TGDF_{ij,4}$ K = 1.2	$TGDF_{ij,4}$ K = 1.4	$TGDF_{ij,4}$ K = 1.6
1-6	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
6-2	0.9275	0.9725	1.0000	0.0000	0.0000	0.0000
1-7	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
7-3	0.2164	0.4041	0.5192	0.0000	0.0000	0.0000
2-8	0.0000	0.0000	0.0000	0.5166	0.6119	0.7591
8-4	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
2-9	0.7146	0.7964	0.8527	0.1185	0.1108	0.1118
9-5	0.7146	0.7964	0.8527	0.1185	0.1108	0.1118
3-10	0.0000	0.0000	0.0000	0.9319	0.9689	1.0000
10-4	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
4-11	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
11-5	0.0000	0.0000	0.0000	0.9131	1.0000	1.0000

TABLE VI
LGDFs for the Reactive Power for Different System Loading K

Line i-j	$LGDF_{ij,1}$ K = 1.2	$LGDF_{ij,1}$ K = 1.4	$LGDF_{ij,1}$ K = 1.6	$LGDF_{ij,4}$ K = 1.2	$LGDF_{ij,4}$ K = 1.4	$LGDF_{ij,4}$ K = 1.6
1-2	0.8007	0.8520	0.8832	0.0000	0.0000	0.0000
1-6	0.8007	0.8520	0.8832	0.0000	0.0000	0.0000
1-3	0.8007	0.8520	0.8832	0.0000	0.0000	0.0000
1-7	0.8007	0.8520	0.8832	0.0000	0.0000	0.0000
2-4	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893
8-4	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893
2-5	0.6432	0.7255	0.7801	0.1671	0.1293	0.1038
2-9	0.6432	0.7255	0.7801	0.1671	0.1293	0.1038
3-4	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893
10-4	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893
4-5	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893
4-11	0.0000	0.0000	0.0000	0.8497	0.8711	0.8893

Appendix

For the generator k, it is possible to calculate total line power flow S_{ij} on the line i-j as follows:

$$S_{ij} = \sum_{k=1}^{r} d_{ij,k} G_k \quad \text{for } j \in \Xi_i \,, \tag{32}$$

where r is the total number of generators sharing the line load. Equation (32) can be reformulated using (7) as:

$$S_{ij} = \sum_{k=1}^{r} LGDF_{ij,k} S_{ij} \quad \text{for } j \in \Xi_i \,. \tag{33}$$

Similarly, the line power flow S_{ji} at the other side of the line i-j can be calculated as:

$$S_{ji} = \sum_{k=1}^{r} LGDF_{ij,k} S_{ji} \quad \text{for } j \in \Xi_i \,, \tag{34}$$

thus the transmission losses L_{ij} at the same line can be formulated as:

$$L_{ij} = \sum_{k=1}^{r} LGDF_{ij,k} S_{ij} - \sum_{k=1}^{r} LGDF_{ij,k} S_{ji} \quad \text{for } j \in \Xi_i \,, \tag{35}$$

that leads to the following notation:

$$L_{ij} = \sum_{k=1}^{r} LGDF_{ij,k} L_{ij} \quad \text{for } j \in \Xi_i \,, \tag{36}$$

where the sum of all LGDFs are equal to 1:

$$\sum_{k=1}^{r} LGDF_{ij,k} = 1 \,. \tag{37}$$

Thus, the transmission losses can be fairy allocated among producers using the line generation distribution factors. Similarly, it can be shown, that transmission losses can be allocated among consumers transparently.

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