Evaluating Generating Unit Unavailability Using Bayesian Power Priors

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Abstract **— Generating unit unavailability assessment is an important task in the power system generation expansion planning aimed at managing an acceptable degree of security of supply. In the case of the newly installed or planned units the sample size of the data for unavailability assessment is limited and inadequate to provide the desired accuracy in the unavailability estimation. A new concept based on the Bayesian power prior approach has been developed to utilize the data from similar generating units. The original contribution of the present work is a model that incorporates the data of unavailability from other generating units into the statistical analysis of unavailability of the analysed generating unit to improve the accuracy of the estimation. The empirical results show that for unavailability estimation, the power prior Bayesian approach exhibits better than the classical statistical approach in both the standard error of estimate and confidence interval as the measures of accuracy.**

Index Terms **—Generating unit reliability, Bayesian inference, informative prior, power generation, unavailability**

I. INTRODUCTION

The deregulation and privatization of the energy sector and growth of renewable electricity generation have brought changes in the way the power system may be developed and operated. In the process of generation expansion planning, one is interested in the probability that the power system will be able to provide sufficient amounts of electric energy and power to cover the consumption and system load. If generating units are not available (e.g. during the outage or maintenance) the ability of the system to provide sufficient amounts of electricity is reduced accordingly. This reduction occurs in both planned and unplanned outages [1].

The basis for power system reliability analysis is the evaluation of reliability of a single generating unit. In order to account for randomly occurring outages, various deterministic criteria are in practical use, e.g. the installed generation capacity equals the expected maximum demand plus fixed percentage of it, the spinning capacity equals the expected maximum demand plus reserve equal to one or more largest units. The drawbacks of deterministic criteria could be in less economic and objective assessments in the decision making process [2]. Generating unit unavailability is often taken as a constant value for the whole planning period. A drawback of that approach is that due to the random nature of failures there will be a substantial variation in annual availability [3]. Probabilistic methods in reliability assessment that can address objectively the stochastic nature of power system behaviour can aid in the generation expansion planning.

Regardless of the fact that the need for probabilistic evaluation was identified some 80 years ago [2], the development of the new models and improvement of the existing ones is very topical in the light of reliability-centred asset management approach [4]. This approach requires good insight into the effect of maintenance on the reliability and costs of power systems. The goal of a statistical reliability analysis is to turn data into information that serves as the basis for various decisions. The process of the statistical reliability analysis is presented in Fig. 1.

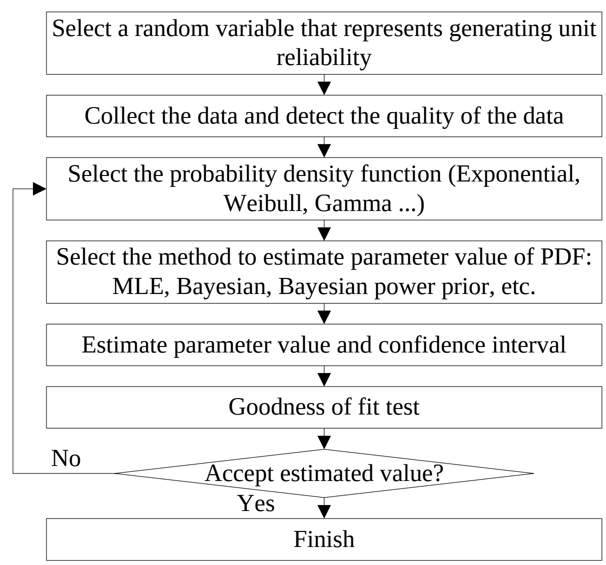

Fig. 1. Process of statistical estimation of parameter values

In the long term power system planning one is often faced with the limited amount of data (small sample size) due to usages of the annual interval (one value per year), "young" or planned elements without history. The small sample size hampers the corresponding evaluation and is often inadequate to provide desired accuracy of the estimated parameters [5]. An effective approach should allow the assessment of the elements' reliability under the limited amount of data.

Strictly statistically speaking, a classical (frequentist) method for assessing the parameters of reliability is acceptable in cases where there are sufficient amount of data [6]. In order to estimate reliability more efficiently under the small sample size some authors combine outage data with the technological

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and physical information, related to wear and stress [7]. The drawback of those approaches is that one needs an insight into the aging properties, wear mechanisms and the relations between them, which is not always available and is often complicated and difficult to apply [8].

Another approach is to pull together data from a number of identical generation units. In [9], the authors use the binomial law to estimate the unavailability of identical wind turbines or PV inverters. In contrast to identical elements, this approach is not suitable for non-identical ones as thermal power plants are usually considered. In [10], the authors group generation units to calculate unit statistics as forced outage hours and equivalent forced outage rate. The unweighted approach is used for time-based methods where all units are considered equal in terms of outage impact. Regarding the fact that we use the time-based unavailability approach, it is not appropriate to pull all generating units into one group due to the differences among these generation units.

The main challenge for the present work is to improve the accuracy of estimation of annual generator unavailability by the incorporation of data from another (similar) generator. In order to use those data as prior information, the Bayesian methodology was developed [11]. Prior information is the information from sources other than the system (in our case a generating unit) under investigation that is available prior to the analysis [12], [13]. Sources of prior information could be experts' knowledge, literature, data from the previous analysis and technological information [7].

However, raw prior information such as data from other sources is often available. For example, in the case of the annual unavailability analysis of a generating unit, data from a similar generating unit are available. In a situation like that, it is natural to incorporate those data into the current analysis. The new approach is based on a concept of the power prior proposed by Ibrahim and Chen [14]. The advantage of the power prior approach over the basic Bayesian approach is to enable the researchers to weight the data from different sources. Ibrahim and Chen [14], Chen et al. [15] and Ibrahim et al. [16] presented how to construct power priors discussed the general conditions for propriety and gave a formal justification of the power prior as an optimal class of informative priors. As a consequence of using more data, the Bayesian power prior method has the advantages in terms of estimation accuracy for decisions under small sample size.

This paper proposes applications of the power prior in the assessment of parameters of the electricity generating unit annual unavailability PDF. The main original contribution of the present work is a Bayesian power prior model that incorporates the data of annual unavailability from other generators into the statistical analysis of annual unavailability of the analysed generator. This model enables the accuracy improvement in the estimation of annual unavailability PDF parameters. The main advantage of the power prior method over using a simple pulling of other data is the ability to control the influence of the data from other sources on the current analysis [17]. The proposed solution thus overcomes the limitation of classical statistics in the case of a limited amount of data. The subjective information about the difference in two generators is incorporated by adjusting the hyperparameters in the prior. We also compare the properties of the Bayesian estimators and the classical estimators in practice by assessing the estimated standard error and the confidence interval as basic comparison criteria.

The paper is organized as follows: In section II, the Bayesian power prior methodology is presented. In Section III, the simulation study and the application of power priors in generation unavailability assessment are demonstrated. Finally, the conclusions are given in Section IV.

II. Statistical Estimation of Parameters

A generating unit as a repairable system, through its lifetime, undergoes alternating in-service times and repair/maintenance times [18]. The general aim of the generation unit annual unavailability assessment is to select PDF and estimate its parameters for the assessed generating unit. Annual time unavailability of generating units is defined as a percentage of the time a system is not functional when needed [19]:

$$x_i = \frac{T_{nv,i}}{T_{N,i}} \tag{1}$$

where i represents year of valuation, while $T_{\mathrm{nv},i} = T_{\mathrm{nvp},i} + T_{\mathrm{nvu},i}$ is unavailable time as a period in which the unit cannot be operated for reasons which are inside the plant, and is composed of planned ($T_{\mathrm{nvp},i}$) and unplanned ($T_{\mathrm{nvu},i}$) parts. $T_{\mathrm{N},i}$ is a reference period as the total recording period (e.g. calendar year).

As a probability model we use the exponential PDF:

$$f_X(x \mid \theta) = \frac{1}{\theta} e^{-\frac{1}{\theta}x} \tag{2}$$

where x is annual unavailability, and parameter θ represents an expected value of annual unavailability:

$$\theta = E(X) \tag{3}$$

where the expected value $E(x)$ is calculated as follows:

$$E(X) = \int_{-\infty}^{\infty} x f_X(x \mid \theta) dx \tag{4}$$

If X is a random variable with the expected value $E(X)$, the variance of X is:

$$Var(X) = E\{[X - E(X)]^2\} \tag{5}$$

provided that the expectation exists. The standard deviation of X is the square root of the variance:

$$\sigma = \sqrt{Var(X)} \tag{6}$$

The most widely used PDFs in the reliability analysis are Exponential, Weibull, Gumbel and Gamma. The exponential distribution is appropriate since it is assumed that the power plant does not undergo any degradation over time and consequently the failures are likely to occur equally over time, [18]. .

In order to fit a probability model to data, we have to estimate parameters associated with the probability model from the data. The results of unknown parameter estimation are: point estimate or one value of parameter, standard error of point estimates and confidence or the Bayesian high posterior density interval of estimates [5].

In the following subsections the Bayesian power prior inference is developed for the purpose of generating unit unavailability estimation. Prior to that, the method of maximum likelihood and the traditional Bayesian inference method using the informative reference prior elicited from the literature, were presented.

A. Maximum Likelihood

The method of maximum likelihood estimate (MLE) is one of mostly used methods in the classical or frequentist approach to statistics [5]. The basic assumption of this method is that the observed data are most likely or probable. Suppose that random variables X_1, $X_2,\dots,X_n$ have a joint density or frequency function. Given the observed values $X_i = x_i$, $i = 1,\dots,n$, the likelihood as a function of θ is a joint density function and is defined as a product of marginal densities [5]:

$$L(\mathbf{x} \mid \theta) = \prod_{i=1}^{n} f_X(X_i \mid \theta) \tag{7}$$

where vector $\mathbf{x} = (x_1, x_2, \dots, x_i, \dots, x_n)$ is the vector of n observed values. The likelihood function $L(\mathbf{x}|\theta)$ is considered as function of θ, because x_i's are the already observed values. MLE of θ is that value of θ which maximizes the likelihood function as:

$$\theta_{est} = \arg\max_{\theta}\left[L(\mathbf{x} \mid \theta)\right] \tag{8}$$

In order to obtain the standard error (SE) $s(\theta_{\text{est}})$ and confidence interval (CI) of MLE, we use the bootstrap method. Bootstrapping is the practice of estimating standard error of θ_{est} by measuring it when sampling from the selected PDF. We generate 1000 samples of size n from the exponential PDF with parameter value estimated from input data θ_{est}. From each of these samples we calculate new estimates of θ. The variability of the θ_{est} can be summarized by calculating the standard deviations of the 1000 estimates of θ, thus providing estimated standard errors of θ_{est} as:

$$s(\theta_{est}) = \sqrt{\frac{1}{1000}\sum_{i=1}^{1000}\left(\theta_i^* - \bar{\theta}\right)^2} \tag{9}$$

where $\bar{\theta}$ is the mean of the 1000 estimated values of θ, while θ_i^* is 1000 estimates of θ. The CI for θ_{est} is a random interval that contains θ_{est} with some specified probability. A more detailed description of the MLE and bootstrap is available in [5].

B. Bayesian Inference

Bayesian inference is built on Bayes' rule as a fundamental mathematical ingredient of a subjective, or "Bayesian", approach to epistemology, theories of evidence, and theories of learning [5]. According to that, knowledge from data other than the current random sample (e.g. individual's beliefs) about the parameter value or *a priori* knowledge can be coded in probabilities.

Bayes' rule defines the change in probability of an event P(A) after another event B occurs. The probability assessment of event A is revised in the face of evidence of event B according to Bayes' theorem:

$$P(A \mid B) = P(A)\frac{P(B \mid A)}{P(B)} \tag{10}$$

Bayes' rule could be used in the estimation of parameters of the probability model that generated data. Suppose that given a parameter θ, the random variable X follows a distribution with density $f_X(x|\theta)$. Bayesian analysis is performed by combining the prior information of parameter θ and the data x_i into the posterior distribution of θ given x_i. The basic assumption in Bayesian analysis is that parameter θ is a random variable (in contrast with the frequentist statistics where parameter θ is treated as an unknown constant) with probability density function $\pi_\Theta(\theta)$, denoted the prior of θ. This assumption represents a basic difference between Bayesian and classic (frequentist) statistics.

Furthermore, the posterior distribution of θ given $\mathbf{x}$, denoted $\pi_\Theta(\theta|\mathbf{x})$ is the conditional distribution of θ given the sample observations x_i. Applying Bayes' rule on the random variable, we have:

$$\pi_\Theta(\theta \mid \mathbf{x}) = \pi_\Theta(\theta)\frac{L(\mathbf{x} \mid \theta)}{f_X(\mathbf{x})} = \pi_\Theta(\theta)\frac{L(\mathbf{x} \mid \theta)}{\int_\Theta \pi_\Theta(\theta)L(\mathbf{x} \mid \theta)d\theta} \tag{11}$$

where $\pi_\Theta(\theta)$ is the prior PDF of parameter value θ, $L(\mathbf{x}|\theta)$ is the likelihood function of parameter value θ and $f_X(\mathbf{x})$ is the marginal probability of X. Here, Θ is the random variable with values θ. An equivalent form of (11) omits the factor $f_X(\mathbf{x})$. With fixed $\mathbf{x}$ (already observed values) the denominator in (11) does not depend on parameter value θ because the integral in denominator is integrated over the whole sample space Θ of values θ. The posterior distribution can be written as:

$$\pi_\Theta(\theta \mid \mathbf{x}) \propto L(\mathbf{x} \mid \theta)\pi_\Theta(\theta) \tag{12}$$

where symbol $\propto$ denotes "proportional to".

Posterior distribution $\pi_\Theta(\theta|\mathbf{x})$ reflects the updated information about θ after the data $\mathbf{x}$ are observed. It expresses uncertainty about parameter θ after taking both the prior and the observed data into account. Based on posterior distribution, a decision maker can make e.g. point estimation inference on θ, confidence interval of θ.

The information needed to estimate Θ is contained in the posterior distribution $\pi_\Theta(\theta|\mathbf{x})$. In order to estimate a parameter value θ, a posterior mean or posterior mode could be used. A Bayesian analogue of MLE SE is the posterior standard deviation [5]. CI in Bayesian analysis is the interval between two selected percentiles (e.g. from the 5^{th} to the 95^{th} percentile) of the posterior. An alternative to this interval is a high posterior density (HPD) that is defined as the narrowest interval, which for a unimodal distribution will involve choosing those values of highest probability density including the mode [5].

C. Informative Prior Elicitation

In Bayesian inference, a prior probability distribution of an uncertain parameter θ is a probability distribution that expresses uncertainty about θ before the data are considered [5]. The parameters of a prior distribution are called hyperparameters, to distinguish them from the parameters (Θ) of the model. Prior probability distributions have traditionally belonged to one of two categories: non-informative and informative priors. One uses a non-informative prior in a case when knowledge about an unknown parameter does not exist prior to data are available. Contrary to a non-informative prior, an informative prior expresses specific, definite information about a random variable. It could be elicited from the experts' knowledge, literature review, explicitly from an earlier data analysis or from different combinations of available information.

Informative prior elicitation is one of the most important issues in the Bayesian inference and is very challenging to practitioners [20]. In the presence of historical data, the power prior method could aid because of a much more systematic approach. In such cases, the quantification of prior information could be more objective than experts' knowledge or literature elicitation.

D. Bayesian Power Prior Analysis

Suppose that θ is the parameter of PDF of generating unit annual unavailability and $L(\theta|x_0)$ is the likelihood function of θ based on the historical data or data from other sources, denoted by $\mathbf{x}_0$. We assume that, given θ, data from other sources ($\mathbf{x}_0$) and current data ($\mathbf{x}$) are independent random samples from an exponential PDF. $\pi(\theta)$ is defined as the non-informative initial prior. Given a_0, Ibrahim and Chen ([14]) define the power prior of θ for the current analysis as:

$$\pi_\Theta(\theta \mid \mathbf{x}_0, a_0) \propto L(\theta \mid \mathbf{x}_0)^{a_0} \pi_0(\theta) \tag{13}$$

where power prior $\pi_\Theta(\theta|\mathbf{x}_0,a_0)$ is proportional to the initial prior $\pi_0(\theta)$ and the likelihood function of data from other sources raised to the power parameter a_0, also named hyperparameter. Given power prior (13), posterior PDF was defined as [14]:

$$\pi_\Theta(\theta \mid \mathbf{x}, \mathbf{x}_0, a_0) \propto L(\theta \mid \mathbf{x}) L(\theta \mid \mathbf{x}_0)^{a_0} \pi_0(\theta) \tag{14}$$

where posterior PDF $\pi_\Theta(\theta|\mathbf{x},\mathbf{x}_0,a_0)$ is proportional to the power prior and the likelihood function of data from the analysed unit.

The parameter a_0 defines the weight of likelihood of data from another source ($\mathbf{x}_0$) chosen in the current study. The likelihood is defined using the prior in (13). The case $a_0 = 0$ implies that no data from other sources should be used, while $a_0 = 1$ gives equal weight to $L(\theta|\mathbf{x}_0)$ and the likelihood of the current data $L(\theta|\mathbf{x})$, resulting in full incorporation of the data from other sources. Therefore, (12) can be viewed as a generalization of the usual Bayesian update of $\pi_\Theta(\theta)$ [14]. The power parameter a_0 can be interpreted as a precision parameter because smaller a_0 implies larger power prior variance, while larger a_0 means smaller power prior variance [21].

The power prior parameter a_0 could be elicited as a deterministic or random number. The advantage of a deterministic parameter is that we know exactly how much historical data $\mathbf{x}_0$ are incorporated into the analysis [22]. In order to choose between deterministic or random a_0, experience shows that taking a_0 as a deterministic number is more computationally feasible and easier to interpret than taking a_0 as a random variable [23]. We therefore use deterministic a_0 with several sensitivity analyses. They are important because data from another source may cause a biased estimate of parameters. In situations like that, experts' knowledge and experiences could aid to a choice of the most appropriate estimate of parameters. An expert could determine a degree of overlap between the analysed and other generators based on the unit's characteristics. In practice, we should interview more than one expert to get a more objective estimation of a_0. In the case of discrepancies, a sensitivity analysis should be performed. We consider expert methods for determining the similarity of two generation units. In our case, important criteria are: the same fuel, site, environment and maintenance practice.

E. The Choice of a_0

To address the issue of selecting a_0, we use the value a_g that yields the best model fit by minimizing to the marginal likelihood criterion taking the form [23]:

$$G(a_0) = -2\log[m(a_0)] \tag{15}$$

where $m(a_0)$ is:

$$m(a_0) = \frac{\int L(\theta \mid \mathbf{x}) L(\theta \mid \mathbf{x}_0)^{a_0} \pi_0(\theta) d\theta}{\int L(\theta \mid \mathbf{x}_0)^{a_0} \pi_0(\theta) d\theta} \tag{16}$$

Thus, the minimizer to (15) is a_g. As is stated in [23] and [16], the criterion (15) appears to work quite well in practice for determining a_g. It is emphasized that the guide value derived from (15) serves only as a starting point for the analysis. Ibrahim et al. [16] recommend making several analyses in the range of the guide value. They do not recommend making an analysis based on a single a_0 value or using a one-time automated procedure [16].

The advantage of the power prior method is in improving the annual unavailability estimation accuracy by using information from other generating units [21]. Besides the marginal likelihood criterion we also use the value a_s that yields the best model fit by minimizing the posterior standard deviation [5] named standard error (SE):

$$SE = \sqrt{\sum_{j=1}^{N} \left(\theta_j - \bar{\theta}\right)^2 p(\theta_j)} \tag{17}$$

where N is the number of simulations, θ_j is the value of the selected estimate on the j-th simulations, and

$$p(\theta_j) = L(\theta_j \mid \mathbf{x}) L(\theta_j \mid \mathbf{x}_0)^{a_0} \pi_0(\theta_j) \tag{18}$$

is the posterior distribution function of θ_j.

The minimum posterior SE guided value of a_0 is that value of a_0 that minimizes the posterior standard deviation as:

$$a_s = \arg\min_{a_0} [SE] \tag{19}$$

To obtain parameter estimates for (15) and criterion (16) and (18), where integration is difficult to obtain by elementary means or evaluate using tables of integrals, the posterior probability density and likelihood functions are numerically integrated by Monte Carlo integration methods.

Integrals used in (16) take the form:

$$I = \int_a^b h(\theta) \pi_0(\theta) d\theta \tag{20}$$

If $\pi_0(\theta)$ is probability distribution, integral I could be restated as:

$$I = E_{\pi_0(\theta)}[h(\theta)] \approx \frac{1}{N} \sum_{j=1}^{N} h(\theta_j) \tag{21}$$

This statement says that if we can sample values of θ using $\pi_0(\theta)$, then the value of the original integral I is simply a scaled version of the expected value, approximated by the sample mean of the integrand function $h(\theta)$ calculated using those samples where samples θ_j, j=1, 2,…, N, are drawn independently from $\pi_0(\theta)$. This leads to a simple four-step procedure for performing Monte Carlo approximation to the integral I: (1) identify $h(\theta)= L(\theta|\mathbf{x}_0)^{a0}$ or $h(\theta)= L(\theta|\mathbf{x})\cdot L(\theta|\mathbf{x}_0)^{a0}$ respectively; (2) identify $\pi_0(\theta)$; (3) draw N independent samples from $\pi_0(\theta)$; (4) evaluate (17). A more detailed description is available in [5].

F. Power Prior Application in Generator Unavailability

When applying the Bayesian analysis with power priors to generating unit annual unavailability data, two kinds of additional information could be incorporated: historical information or information from a similar generating unit. In this paper we use information from the older generating unit which is located in the same power plant.

Since the two generating units are located in the same power plant and use the same primary source (lignite), the same operational and maintenance practices suggest the complementarily of the units' availability. So the data collected at one unit should partially reflect the availability of another. The annual unavailability data collected for the analysed unit are treated as the current data, while the annual unavailability data collected at the older unit are referred to as "historical" data.

III. Numerical Examples

A. Simulation Study

We have carried out a simulation study to examine the empirical performance of the power prior with a_0 for the exponential probability distribution which is denoted as in (2). The exponential probability model for the current data is $X \sim \exp(\theta)$ for $i = 1,\ldots, n$, and the model for the data from other sources or historical data is $X_0 \sim \exp(\theta_0)$ for $i = 1,\ldots, n_0$, where the X and X_0 are independently distributed random variables, e. g. yearly unavailability. We also assume that the X and X_0 are independent. We considered 6 scenarios as follows: the data from another source X_0 are generated from an exponential model with $\theta_0 = 12$ and $n_0 = 100$; the current data X are generated from exponential models with $\theta = 8, 10, \ldots, 18$ and $n = 100$.

We generated N=1000 simulated datasets under each scenario. For each simulated dataset, we computed the posterior maximum and the posterior SD for θ using the power prior (13) with eleven a_0 values ranging from 0 to 1 with an increment of 0.1 and estimated guided a_0 values, namely, a_g and a_s given by (15) and (19) respectively. In all cases, an improper initial prior distribution, $\pi_0(\theta) \sim$ Unif $[a, b]$ was specified. The method was developed in MATLAB using the MATLAB Statistics and Machine Learning Toolbox [24].

The simulation results are shown in Table I. First, we observe that as θ is varied, weight a_g is varied accordingly. Table I further shows the behaviour of a_g as θ differs from θ_0. We see that larger values of a_g are obtained with a smaller difference between θ and θ_0 with constant n and n_0. In those cases the historical data are particularly down-weighted. Thus the criterion for obtaining a_g ensures that other data cannot dominate or have more influence than the current data.

These results are illustrated in Fig. 2 which shows a plot of the expected value of the criterion function (19) plotted for several values of θ, assuming $n = n_0 = 100$. To show graphs of all simulations on the same plots we have normalized the criterion function which does not influence the minimum value. From Fig. 2 we could see that the smaller the difference between θ and θ_0, the larger a_g. This implies that as the probability law that generated X becomes more like the law that generated X_0, more historical data are incorporated into the analysis. Moreover, in Fig. 2 we see how the convexity of (15) changes as the difference between θ and θ_0 increases.

TABLE I
Simulation Study Results For $\theta_0 = 12$ and $\theta = (8, 10, \ldots, 18)$

θ	8	10	12	14	16	18
a_g	0	0,1	0,9	0,1	0	0,1
$E(SD(a_0{=}1))$	2.22	1.46	1.17	1.47	2.32	3.48
$E(SD(a_0{=}a_g))$	1.12	1.33	1.19	1.75	1.97	2.15
$E(SD(a_0{=}0))$	1.12	1.38	1.65	1.86	1.97	2.11
a_s	0	0,2	1	0,9	0.2	0

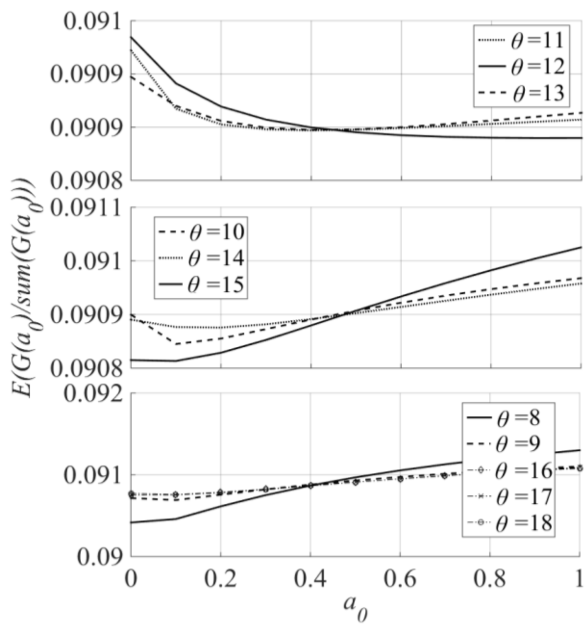

Fig. 2. Normalized criterion $G(a_0)$ averaged for 1000 simulations for several values of θ ($n = n_0 = 100$; $\theta_0 = 12$)

It should be noted that the optimal a_0 (a_g at minimal $G(a_0)$) is close to 1 and, consequently, larger weight should be put on the historical information in the cases when θ is close to θ_0 (top diagram; θ is between 11 and 13). It appears that in the situations when θ is far from θ_0 (mid and bottom diagram) the optimal a_0 approaches zero and, consequently, smaller weight is put on the historical information. Based on the diagrams above, we can conclude that criterion (15) appears to work quite well in practice for determining a_g.

For each simulation, an optimal value a_g has been calculated. Histograms of a_g for different values of θ are shown in Fig. 3. From the figure we can see that the smaller the difference between θ and θ_0, more values of a_g are closer to 1 and more historical data are incorporated into the analysis while the larger that difference is, more values of a_g are closer to 0.

The posterior SE has been estimated for each individual simulation as standard deviation of posterior density (18). Fig. 4 shows the effect of choosing different values of a_0. The SEs (for a_0 and θ) are averaged for the 1000 simulations at each condition. The number of simulations is chosen experimentally as a trade-off between accuracy and execution time. They are represented as a function of a_0 and θ.

As can be seen in the figure, when θ is very different from θ_0 (i.e. when using $\theta = 8$ or $\theta = 18$) the SEs are minimal in the case when no data from other sources are applied ($a_0 = 0$). On the other hand, when the current and the other data are generated from the same probability law ($\theta = \theta_0 = 12$), minimal error is gained by using all data from other sources ($a_0 = 1$).

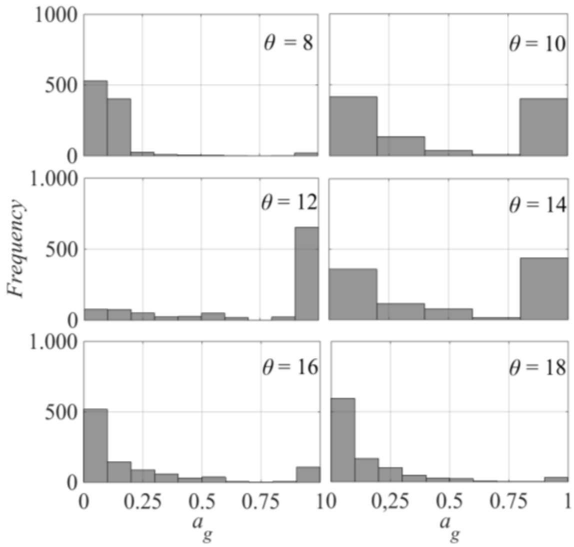

Fig. 3. Histograms of a_g for several values of θ ($n = n_0 = 100$; $\theta_0 = 12$)

The simulation study shows the practical usage of the power prior approach in the case when the current data and those from other sources are generated from the different probability laws. As stated in Introduction, the main advantage of the power prior approach over simple pulling of data from different sources into one set (as is used in classical statistics) is researcher's ability to control the influence of the data from other sources on the current analysis through hyperparameter. From Table I, it can be seen that accuracy of the estimate is improved (SE is lower), if we incorporate appropriately weighted data from other sources ($a_0 = a_g$) even if the data are generated from the probability law with different parameter value.

If the data from a different source contain bad data named outliers, they impact on the probability distribution function. Distribution is skewed in a direction of the outliers. It is expected that the method will weight down the influence of such data set on the analysed one. Regardless of the fact that the method itself can manage bad data, in practice it would be valuable to detect the presence of outliers before the analysis is performed [5].

B. Case Study

The proposed method has been applied to the generating unit annual unavailability probability parameter estimation. We examined two generation units in the Šoštanj Thermal Power Plant (TPP). TPP Šoštanj is the largest fossil fuel power plant in Slovenia, and it accounts for about 33% of the country's generation of electricity [25]. The fuel used for power generation at the plant is lignite which comes from the nearby coal mine. The construction of the power plant began in 1947, and in 1973 the "TEŠ 4" unit began to generate electricity with the net capacity of 275 MW. Five years later the "TEŠ 5" unit with the net capacity of 355 MW was in full operation. The goal of the analysis is to estimate annual unavailability of the "TEŠ 5" unit.

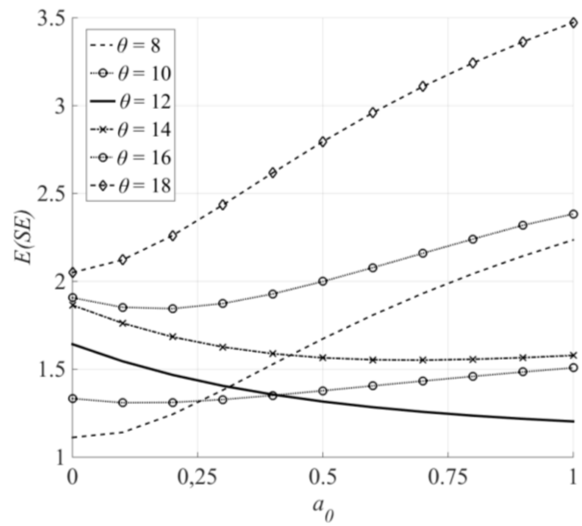

Fig. 4. Posterior standard error SE(a_0,θ) averaged for 1000 simulations as a function of the a_0 for several values of θ ($n = n_0 = 100$; $\theta_0 = 12$)

Regarding the publicly available data from 1973 to 2011 for "TEŠ 4", and from 1978 to 2011 for "TEŠ 5", the sample size of the annual unavailability for "TEŠ 4" is $n_0 = 39$ and for "TEŠ 5" is $n = 34$. In order to grasp brief characteristics of the annual unavailability, some descriptive statistics are shown in Table II.

TABLE II
ANNUAL UNAVAILABILITY OF GENERATING UNITS [%]

Unit	Mean	SD	Min	Max	Years
"TEŠ 4"	12.65	12.91	0.1	41.5	39
"TEŠ 5"	10.60	9.40	0.27	35.1	34

The annual unavailability frequencies on the generating "TEŠ 4" and "TEŠ 5" units are as shown in Fig. 5. From these plots we could see that, compared to TEŠ 4, there is a shift in annual unavailability of TEŠ 5 towards lower values, which implies that TEŠ 5 is more available than TEŠ 4. The majority of outages in both units were caused by the failures of the wastewater treatment unit, boiler leakage and the failure of a slag transport system. Some problems were also caused by bad quality of lignite and difficulties on electrical parts like transformers and the excitation system [26]. The exact cause for higher availability of "TEŠ 5" remains unknown.

We have demonstrated the power prior Bayesian analysis of annual unavailability of "TEŠ 5" using "TEŠ 4" as historical data incorporated via the power prior. In addition, using (15), we computed a_g for this analysis and presented the results based on this value. The units' annual unavailability data were obtained from TPP Šoštanj Operational Bulletin [26]. The unavailability data were taken for years 1973 – 2011. Because the "TEŠ 4" unit is older than the "TEŠ 5" unit, using "TEŠ 4" as historical data may help improve accuracy in the unavailability of “TEŠ 5”.

Suppose that for an individual unit we let n denote the number of years in the analysis. Further, we assume that annual unavailability has an exponential distribution. We let x_i denote the random annual unavailability in the ith year. The variables X_i, $i = 1, 2, ...,$, n are assumed to be independent and identically distributed random variables with a common distribution function (18) that is independent of n. To test how realistic the assumption is, the autocorrelation is computed [5]. For both units the autocorrelation coefficient is significant only in one leg (for unit 4 in the second leg; for unit 5 in the fifth leg), thus it is eligible to assume that the autocorrelation inside the data sets is negligible.

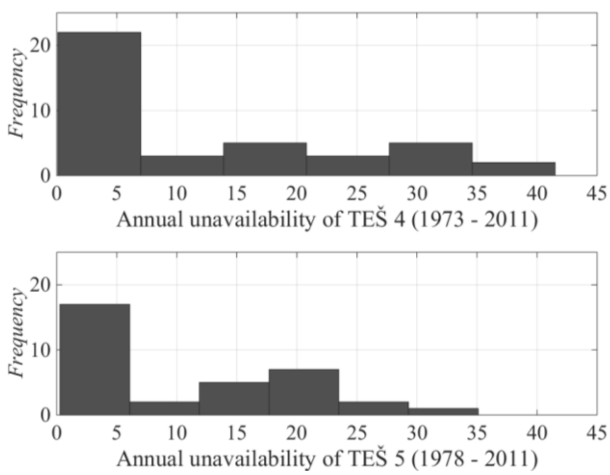

Fig. 5. Histograms of annual unavailability frequencies.

We have used four models to fit the data and computed estimates for existing parameters:

a. Maximum likelihood estimation (MLE),
b. Bayes inference with a prior elicited from the literature (BI),
c. Power prior Bayes inference with a non-informative initial prior (PPBI_NIP) and
d. Power prior Bayes inference with an informative initial prior (IIP) elicited from the literature as in (b) (PPBI_IP).

The algorithm was run for 1000 iterations after the initial 20 iterates were discarded as a burn-in. We have computed the posterior means and 95% highest posterior density (HPD) intervals of the parameter θ. The power prior Bayesian approach (PPBI) has been compared with the frequentist approach (MLE) and classical Bayesian inference (BI). In particular, a uniform initial prior has been assumed for the PPBI_NIP model. For the BI and the PPBI_IP models the normal prior is assumed $\theta_{\text{prior}} \sim N(13.31, 1.72)$. The normal prior has been elicited from the unavailability of fossil fire units in Austria, the Czech Republic, Germany, France, Ireland, Italy, the Netherlands and Poland, available in [27].

The numerical values are reported in Table III. We have noticed that using the power prior based on (18) along with (13), the guide value for a_0 is estimated as $a_g = 1$. This implies that the data from the “TEŠ 4” unit have very much impact on the estimated unavailability of the “TEŠ 5” unit. Table III shows estimates of θ, the standard deviations (SD) also called estimated standard error of estimate (SE), 95% highest posterior density (HPD) intervals or 95% confidence interval in the case of MLE for θ_{est}.

These results are illustrated in Fig. 6, which shows a likelihood of the parameter θ by MLE and posterior PDFs for different models. We can see that intervals obtained by the Bayesian approach are narrower than those obtained by the MLE.

TABLE III
CASE STUDY RESULTS FOR DIFFERENT MODELS

Model	θ_{est}	SE	SE % drop	CI/HPD	CI/HPD % drop
MLE	10.59	1.82	-	[7.32, 14.36]	-
BI_IP	12.20	1.42	21,98	[9.65, 15.10]	22.58
PPBI_NIP(a_0=1)	11.71	1.35	25,82	[9.38, 14.87]	22,02
PPBI_IP(a_0=1)	12.36	1.22	32,97	[10.33, 14.68]	38,21

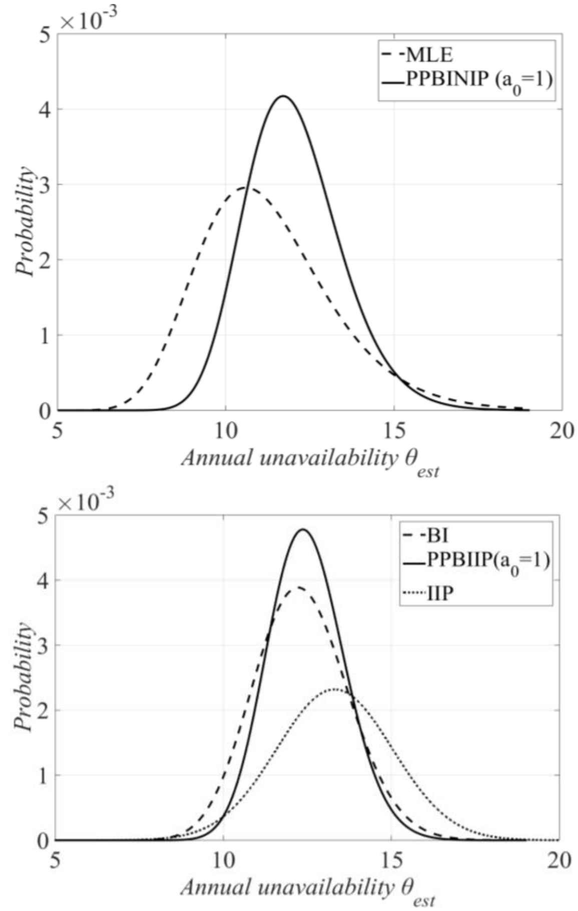

Fig. 6. Probability of (up) MLE, PPBI_NIP, (down) BI, PPBI_NIP, IIP.

C. Comparative Discussion

The case study results presented in Table III reflect some properties of the four estimation models under discussion. As a measure of dispersion of θ_{est}, the standard deviation of θ_{est} is used, also called the standard error of θ_{est}. In classical statistics, a spread of θ_{est} is inversely proportional to the square root of the sample size *n*, while in Bayesian statistics, it is proportional to the prior distribution [5].

As a representative of the classical statistics, the MLE method was considered, while for Bayesian statistics, the basic Bayesian approach (BI), the power prior Bayesian approach (PPBI_NIP), and a combination of both (PPBI_IP) were considered. Table III derives the conclusion that with each source of data added, SE is lower. MLE uses data from TEŠ 5 only; BI_IP uses data from the literature [27] in order to estimate the unavailability of TEŠ 5; PPBI_NIP uses data from "TEŠ 4" to estimate the unavailability of "TEŠ 5"; PPBI_IP uses data both from "TEŠ 4" and from the literature to estimate the unavailability of TEŠ 5. Regarding the fact that the current data consist of only 34 values, it is not surprising that there is a substantial improvement in accuracy (lower SE and narrower CI/HPD) between the approaches.

Table III shows the percentage changes in SE and the width of confidence interval from the MLE estimate using the three estimation models. It can be seen that the Bayesian estimations (BI, PPBI_NIP and PPBI_IP) have substantial reduction of SE and width of confidence interval. It appears that with incorporating the data from the "TEŠ 4" unit (model PPBI_NIP(a_0=1)), SE drops by approximately 25%. After the incorporation of additional data from the literature (model PPBI_IP(a_0=1)), the error drops additionally by 7%. This indicates that it is possible to increase the accuracy under small sample size by utilizing data from other generating units and/or data from the literature. This approach is even more useful in cases where the sample size is very small, which causes high uncertainty and unreliability of the estimated value of the mean annual unavailability (e.g. in the unavailability analysis of a five-year old unit where only 5 values are available). Using the Bayesian power prior approach, the researchers do not need to continue data collection to obtain a larger sample that would be required by classical reliability analysis. Both cost and time savings are achieved as a result of the improved accuracy.

For practical application, an important feature of the proposed method applying the power prior is the influence of the amount of prior information on the model change and on the result. Because the information contained in the data from other generators may be inappropriate for the analysed one, the extreme care must be taken when incorporating those data into the analysis. In [16], the authors analyse optimal properties of the power prior. In situations when the data sample size from other units is substantially larger than the current data sample size, the optimal value of a_g decreases, which means that other data are weighted down. Thus the criteria for obtaining a_g ensure that other data cannot have more influence than the current data.

IV. Conclusions

This paper presents a novel method for the estimation of reliability assessment of generating units; in particular annual unavailability. The reliability model can be estimated with both frequentist and Bayesian approaches. Conventional frequentist approaches may be inappropriate for these data due to small sample sizes. However, we often encounter a situation that data from other sources are available (e.g. experts' knowledge, literature, data from similar generating units etc.). In situations like that, a Bayesian approach can incorporate those data to yield better results. In particular, the Bayesian inference could be applied to make use of data about probability model parameters, while a power prior incorporates historical sample data in a natural way, and it gives the analyst control over the weight given to the historical data through the parameter a_0, which is one of the main advantages of the power prior approach over simple pooling of other data. Since modern power systems are exposed to uncertainties of generating unit outages, the proposed method, which is based on the power prior Bayesian approach, enables better estimation performance by incorporating data from other sources into the statistical inference of annual unavailability.

The method has been tested by both simulation and empirical studies. We have carried out a simulation study to examine the empirical performance of the power prior with a deterministic a_0 for the exponential PDF. The results show that when the current and historical data are generated from similar probability models (e.g. same PDF), the optimal a_0 becomes larger (around 1), and thus more historical data are incorporated in the current PDF estimation. The empirical study performed on two thermal units shows that the incorporation of more data yields more accurate results (standard errors of the estimated parameter are lower). Both studies provide a strong motivation to use the power prior as an informative prior in the Bayesian inference in the generating units' reliability analysis.

Regardless of the fact that the method in this work is applied to annual unavailability, it could also be used in the estimation of other reliability random variables like availability, time-to-failure, time-to-repair etc. Besides that, the method could be used in the estimation of reliability parameters of a newly built or planned unit. One of the possible approaches is to construct an informative initial prior around unavailability factor stated by the manufacturer and incorporate data from other sources, though this remains an issue of future research.

With the availability of data on different power plants, high speed computers and software packages, Bayesian approaches can be used in practice. Analysts could use languages such as R, MATLAB or Phyton, or an application such as WinBUGS. The most appropriate approach to make data available is through sharing the data by a trusted entity through the data repository. Such approach was taken by ENTSO-E with the development of a central information platform which publishes outage information for generation over 100 MW, and can be divided into forced outages and planned maintenance [28]. For each TSO in Europe it is mandatory to submit information related to electricity generation, load, transmission and balancing.

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VI. Biography

Ivan Lorencin was born in Croatia in 1971. In 1997 he gained a bachelor’s degree in Electrical Engineering from the University of Ljubljana, Slovenia, Faculty of Electrical Engineering. He also holds a master’s degree in Information Technology, gained from the University of Ljubljana, Slovenia, Faculty of Computer and Information Sciences. His main research field includes investment in power systems, the power market, and statistics and probability applied to power systems.

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